MOVEMENT OF COFFEE CUP DEFINES A CLOSED PATH IN $SO(3)$

WHERE

$SO(n)$ = GROUP OF $n \times n$ ORTHOGONAL MATRICES WITH DETERMINANT 1

COFFEE MOVES IN $SO(3)$.
IF IT STAYS VERTICAL, IT MOVES IN $SO(2)$

<u>DEF</u> TWO CONTINUOUS MAPS $f_0, f_1 : X \longrightarrow Y$ ARE <u>HOMOTOPIC</u>

IF $\exists$ MAP $h: I \times X \longrightarrow Y$ (WHERE $I = [0,1]$) SUCH THAT

$h(0,x) = f_0(x)$ AND

$h(1,x) = f_1(x)$

$h(0,x) = f_0(x)$ AND

$h(1,x) = f_1(x)$

h IS CALLED A <u>HOMOTOPY</u> BETWEEN f_0 AND f_1.

IT DEFINES A CONTINUOUS DEFORMATION OF f_0 TO f_1.

<u>DEF</u> A HOMOTOPY $\hat{h}$ AS ABOVE IS A PATH IN

$Map(X, Y)$:= SPACE OF CONT. MAP $X \longrightarrow Y$

WITH $\hat{h}(0) = f_0$ AND $\hat{h}(1) = f_1$

$$h(t, x) = \hat{h}(t)(x) \in Y.$$

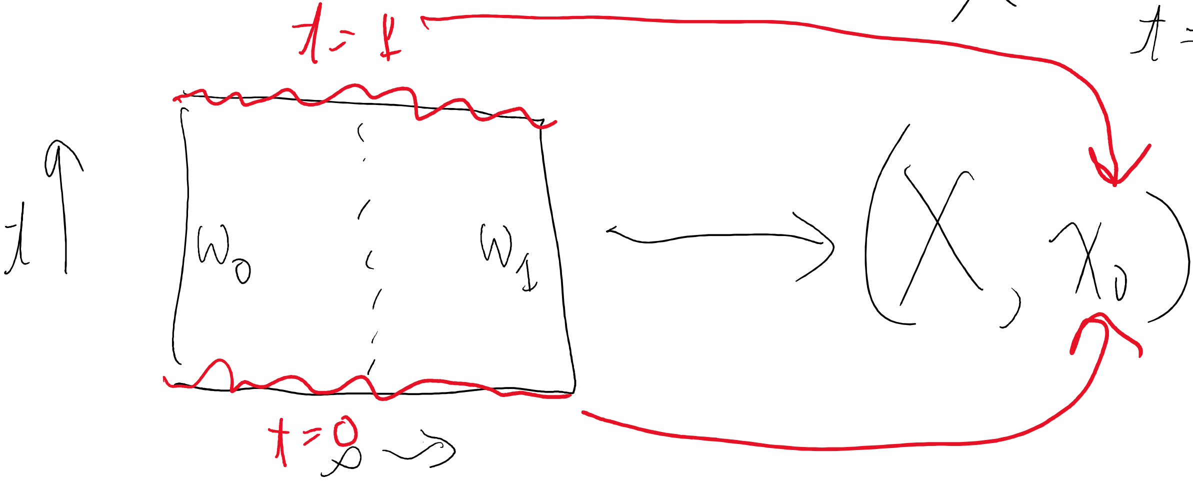
CHOOSE A POINT $x_0 \in X$ AND CONSIDER CLOSED PATH $w: I \longrightarrow X$ WITH $w(0) = w(1) = x_0$

TWO SUCH PATHS w_0; ARE HOMOTO

TWO SUCH PATHS w_0 AND w_1 ARE <u>HOMOTOPIC</u> IF $\exists\ h : I \times I \longrightarrow X$

s = HOMOTOPY PARAMETER

t = PATH PARAMETER



HOMOTOPY LEADS TO AN EQUIVALENCE RELATION AMONG MAPS $X \longrightarrow Y$ OR CLOSED PATHS IN (X, x_0)

<u>THEOREM</u> FOR CLOSED CLASSES IN (X, x_0) THERE IS A BINARY OPERATION ON THE SET OF EQUIV. CLASSES WHICH MAKES THE SET $\pi_1(X, x_0)$ A GROUP

THE SET $\pi_1(X, x_0)$ A GROUP, THE <u>FUNDAMENTAL GROUP</u> OF X AT x_0.

<u>PROOF</u>: LET p, q BE CLOSED