CONTACT
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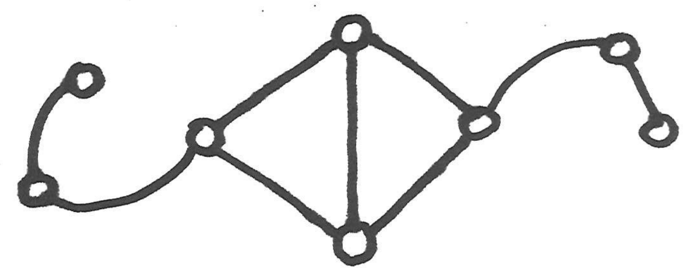
WITH ANY QUESTIONS

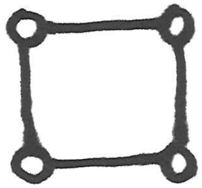
GRAPH THEORY



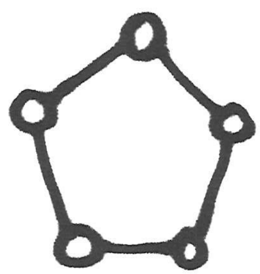
FOR KIDS!

ANOTHER 3D SOLID:

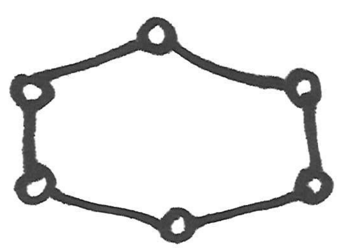
$$\overset{V}{\bigcirc} - \overset{E}{\bigcirc} + \overset{F}{\bigcirc} = \underline{\qquad}$$



$V - E + R =$ ____

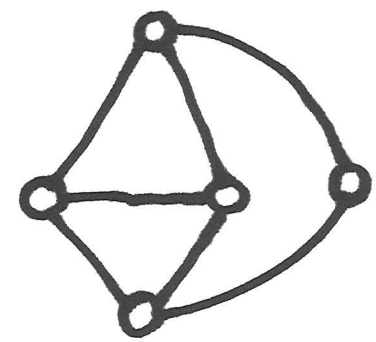


$V - E + R =$ ____



$V - E + R =$ ____

A <u>GRAPH</u> IS A COLLECTION OF VERTICES JOINED BY EDGES.



THIS GRAPH HAS

5 VERTICES

7 EDGES

AND IT DIVIDES THE PLANE INTO

4 REGIONS

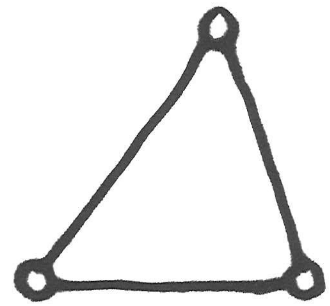
THE MATHEMATICIAN LEONHARD EULER NOTICED SOMETHING PECULIAR WHEN HE CALCULATED:

$$\left(\begin{array}{c}\text{NUMBER OF}\\ \textbf{VERTICES}\end{array}\right) - \left(\begin{array}{c}\text{NUMBER OF}\\ \textbf{EDGES}\end{array}\right) + \left(\begin{array}{c}\text{NUMBER OF}\\ \textbf{REGIONS}\end{array}\right)$$

$$V - E + R$$

THIS NUMBER IS NOW KNOWN AS THE <u>EULER</u> <u>CHARACTERISTIC</u>.

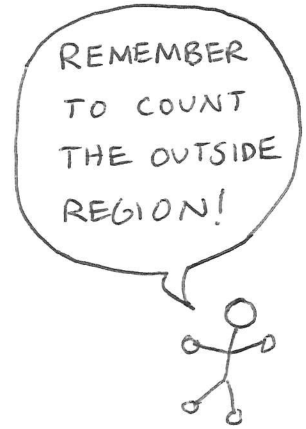
LET'S CALCULATE IT!



VERTICES ______

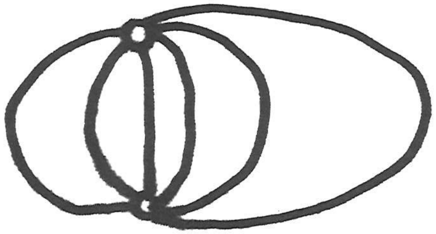
EDGES ______

REGIONS ______

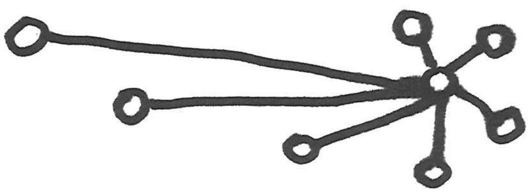


$$\overset{V}{\bigcirc} - \overset{E}{\bigcirc} + \overset{R}{\bigcirc} = \underline{\qquad}$$

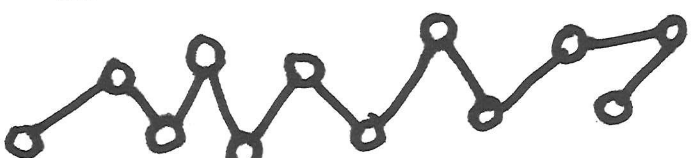
DO WE ALWAYS GET 2?
LET'S TRY SOME EXTREME CASES:



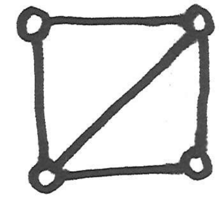
V − E + R = ____



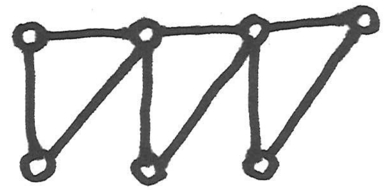
V − E + R = ____



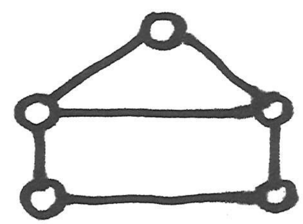
V − E + R = ____



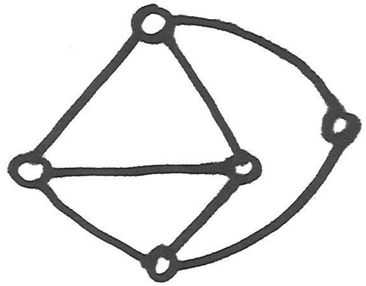
V − E + R = ____



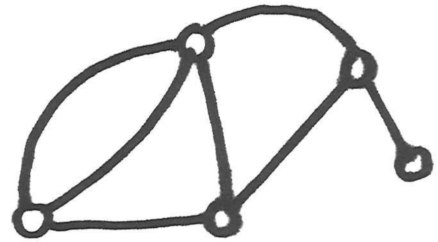
V − E + R = ____



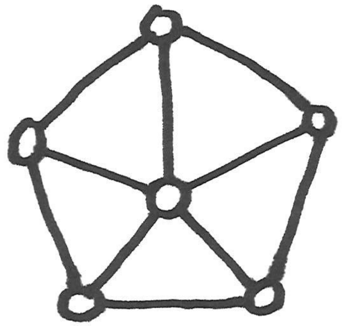
V − E + R = ____



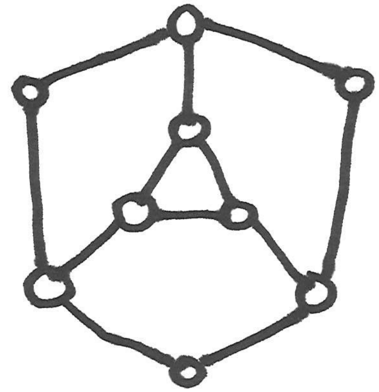
$\overset{V}{\bigcirc} - \overset{E}{\bigcirc} + \overset{R}{\bigcirc} = \_\_\_\_$



$\overset{V}{\bigcirc} - \overset{E}{\bigcirc} + \overset{R}{\bigcirc} = \_\_\_\_$

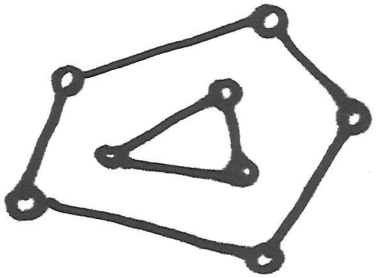


$\overset{V}{\bigcirc} - \overset{E}{\bigcirc} + \overset{R}{\bigcirc} = \_\_\_\_$



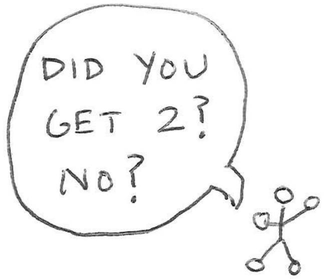
$\overset{V}{\bigcirc} - \overset{E}{\bigcirc} + \overset{R}{\bigcirc} = \_\_\_\_$

LET'S TRY THIS GRAPH:

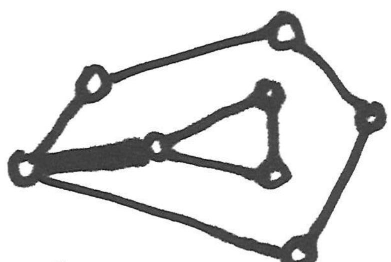


$$V - E + R = \_\_\_$$

THIS GRAPH IS NOT <u>CONNECTED</u>.



WE CAN CONNECT THE TWO COMPONENTS BY ADDING AN EDGE.



$$V - E + R = \_\_\_$$

NOW IT WORKS!

TRY IT YOURSELF!

MY GRAPH:

$$V - E + R = \_\_\_$$

MY GRAPH:

$$\overset{V}{\bigcirc} - \overset{E}{\bigcirc} + \overset{R}{\bigcirc} = \underline{\qquad}$$

MY GRAPH:

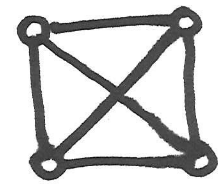
$$\overset{V}{\bigcirc} - \overset{E}{\bigcirc} + \overset{R}{\bigcirc} = \underline{\qquad}$$

CONCLUSION:

EVERY CONNECTED PLANAR GRAPH HAS

$$V - E + R = 2.$$

HOW ABOUT THIS GRAPH?

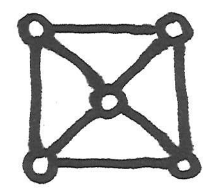


V − E + R = ___

DID IT WORK?

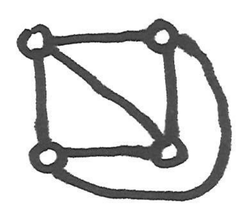
IN THIS GRAPH, THE DIAGONAL EDGES CROSS. LET'S FIX THAT.

ADD A VERTEX



V − E + R = ___

MOVE AN EDGE



V − E + R = ___

NOW IT WORKS!

A <u>PLANAR</u> GRAPH CAN BE DRAWN WITHOUT EDGES CROSSING.

FOR A CONNECTED PLANAR GRAPH, IS THE EULER CHARACTERISTIC ALWAYS 2?

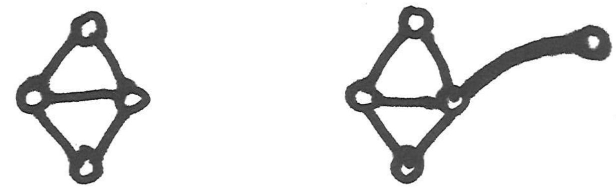
YES!

- IT STARTS OUT TRUE

ONE VERTEX
NO EDGES
ONE REGION

$$\underset{1}{V} - \underset{0}{E} + \underset{1}{R} = 2$$

- IT STAYS TRUE WHEN WE ADD A CONNECTED VERTEX.



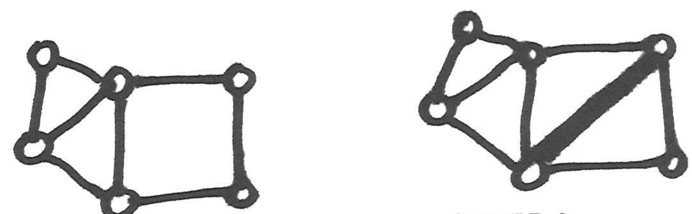
BEFORE AFTER

ONE NEW VERTEX
ONE NEW EDGE

$$V - E + R$$

VERTICES & EDGES BALANCE EACH OTHER

- IT STAYS TRUE WHEN WE CUT A REGION WITH A NEW EDGE.



BEFORE AFTER

ONE NEW EDGE
ONE EXTRA REGION

$$V - E + R$$

EDGES & REGIONS BALANCE

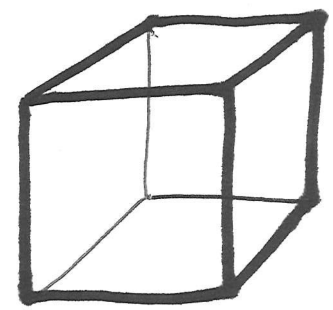
CAN YOU DRAW A 3D SOLID?

MY SOLID:

V - E + F = ____

LET'S CONSIDER THE SURFACES OF SOME THREE-DIMENSIONAL SOLIDS.

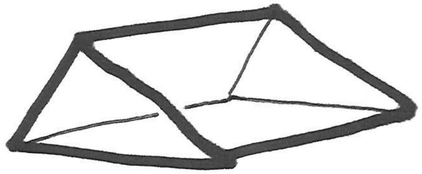
CUBE



VERTICES - EDGES + FACES = ____

WE CALL THEM "FACES" INSTEAD OF "REGIONS".

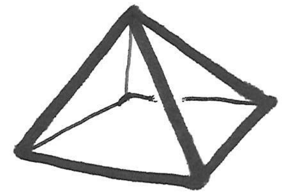
TRIANGULAR PRISM



V E F

○ - ○ + ○ = ____

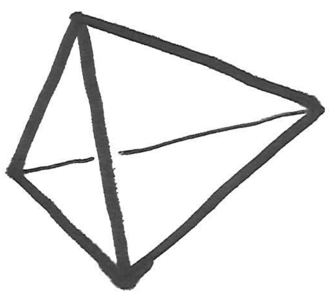
PYRAMID (SQUARE BASE)



V E F

○ - ○ + ○ = ____

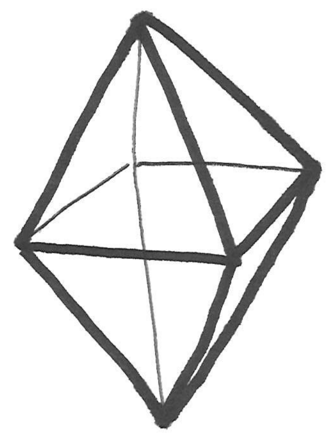
TETRAHEDRON



V E F

○ - ○ + ○ = ____

OCTAHEDRON



V E F

○ - ○ + ○ = ____