MTH 236, Spring 2024 - Homework 10

Due on April 5th at 11:59pm on gradescope

1. Show that $\mathbb{R}\times\mathbb{R}$ and $\mathbb{C}$ are not isomorphic as rings, but that the additive groups $\langle\mathbb{R}\times\mathbb{R},+\rangle$ and $\langle\mathbb{C},+\rangle$ are isomorphic.

2. Let S be the set of matrices $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ with $a, b \in \mathbb{R}$.

   (a) Show that S is a subring of $M_2(\mathbb{R})$.

   (b) Show that S is isomorphic (as a ring) to $\mathbb{C}$.

3. Describe all ring homomorphisms from $\mathbb{Z}$ to $\mathbb{Z}\times\mathbb{Z}$. Which are one-to-one? Onto?

4. An element x in a ring R is called an *idempotent* if $x = x^2$. Let R be a ring in which all elements are idempotents (such a ring is called Boolean). Prove that

   (a) Every element of a Boolean ring R is its own additive inverse. (That is, R has characteristic 2.)

   (b) Every Boolean ring is commutative . [Hint: Look at $(x+y)^2$.]

   (c) Show that a division ring contains exactly two idempotents.