MATH 235: HOMEWORK 6

DUE: SATURDAY, MARCH 2 AT 11:59 PM ON GRADESCOPE
UNIVERSITY OF ROCHESTER, SPRING 2023

Follow the instructions on the course homework page to complete this assignment. Please adhere to the honesty policy detailed on the website. **Justify your answers fully.**
1.

(a) Let $T : P_2(\mathbb{R}) \to \mathbb{R}$ be a linear transformation such that $T(3x^2+4) = 12, T(2x-5) = 11$ and $T(x^2 + x) = -6$. Determine $T(1)$.

(b) Is there a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}$ such that $T(0, 1) = 0, T(1, 2) = 3$ and $T(1, -1) = 2$? Why or why not.

2. Let $T : V \to W$ be a linear transformation, and $v_1, \ldots v_n$ elements of V.

(a) Prove that if T is onto W and that $\{v_1, \ldots v_n\}$ generate V then $\{Tv_1, \ldots Tv_2\}$ generate W.

(b) Prove that if T is $one - to - one$ and $\{v_1, \ldots v_n\}$ is linearly independent, then $\{Tv_1, \ldots Tv_2\}$ is linearly independent.

3. For each of the following transformations, determine the kernel and the range and whether the transformation is one-to-one and/or onto.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (2x - 3y, 5x + y)$.
(b) $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (8x + 4y, 2x + y)$.
(c) $T : \mathbb{R}^3 \to \mathbb{R}^2$, $T(x, y, z) = (x - y, y - z)$.

4. Let B be a fixed $n \times n$ matrix with entries in F, and define $\Phi : M_{n\times n}(F) \to M_{n\times n}(F)$ by $\Phi(A) = BAB^{-1}$.

(a) Show that Φ is linear (Hint: use Theorem 2.10(a) from the book).
(b) Show that Φ is an isomorphism.

5. Suppose V, W are finite-dimensional vector spaces and $T : V \to W$ is an isomorphism. Suppose V_0 is a subspace of V. Show that $T(V_0)$ (that is, the set of all vectors of the form $T(v)$ for $v \in V_0$) is a subspace of W of the same dimension and V_0.