MATH 235: HOMEWORK 3

DUE: FRIDAY, FEBRUARY 9 AT 11:59 PM ON GRADESCOPE
UNIVERSITY OF ROCHESTER, SPRING 2024

Follow the instructions on the course homework page to complete this assignment. Please adhere to the honesty policy detailed on the website.

Problem 1. Suppose S_1 and S_2 are subsets of a vector space V.

(1) Show that if $S_1 \subseteq S_2$, then $\operatorname{span}(S_1) \subseteq \operatorname{span}(S_2)$.
(2) Show that $\operatorname{span}(S_1 \cap S_2) \subseteq \operatorname{span}(S_1) \cap \operatorname{span}(S_2)$.
(3) Show that $\operatorname{span}(S_1 \cup S_2) = \operatorname{span}(S_1) + \operatorname{span}(S_2)$. (Use the definition for the sum of subspaces from Homework 1.)

Problem 2. .

(1) Show that a subset S of a vector space V is linearly independent if and only if every subset of S is linearly independent.
(2) Suppose v, w are vectors in a vector space V. Show that $\{v, w\}$ is linearly independent if and only if neither vector is a scalar multiple of the other.

Problem 3. . Suppose $\{v_1, v_2, v_3, \ldots, v_m\}$ spans V. Does

$$\{v_1 - v_2, v_2 - v_3, v_3 - v_4, \ldots, v_{m-1} - v_m, v_m\}$$

also span V?

Problem 4. Suppose $S = \{v_1, v_2, v_3, \ldots, v_m\}$ is linearly independent in V.

(1) Suppose $\lambda \in F$ and $\lambda \neq 0$. Is $\{\lambda v_1, \lambda v_2, \lambda v_3, \ldots, \lambda v_m\}$ linearly independent?
(2) Suppose $w \in V$. Show that if $\{v_1 + w, v_2 + w, v_3 + w, \ldots, v_m + w\}$ is linearly dependent, then $w \in \operatorname{span}(S)$.

Problem 5. .

(1) Can $P_3(\mathbb{R})$ (the polynomials of degree at most 3 with coefficients in $\mathbb{R}$) have a spanning set containing no vectors of degree exactly 2? Why or why not?
(2) Let $p_k(x) = \sum_{i=0}^{k} x^i$. Show that $\{p_k(x) \mid 0 \leq k \leq n\}$ is a basis for $P_n(\mathbb{R})$.
(3) Let $S = \{1, \cos x, \cos^2 x, \sin^2 x, \cos(2x), x\} \subset \mathcal{F}(\mathbb{R}, \mathbb{R})$. Determine a subset of S that is a basis for $\operatorname{span}(S)$.