Homework 10

Due Saturday, April 6 , 2024 at 11:59pm on gradescope

Academic honesty expectations: Same as on previous homeworks. We remind you that internet searches are not permitted.

1. Compute the determinant of the following matrix by first reducing it to an upper-triangular matrix.

$$A = \begin{pmatrix} 3 & -1 & 5 & 6 \\ 6 & 0 & 8 & 10 \\ 0 & 7 & -5 & -1 \\ -3 & 5 & 0 & 1 \end{pmatrix}$$

2. Let $v = (v_1, v_2, v_3)$ and $v = (v_1, v_2, v_3)$ be vectors in $\mathbb{R}^3$ and consider the column matrix

$$A = (v, u) = \begin{pmatrix} v_1 & u_1 \\ v_2 & u_2 \\ v_3 & u_3 \end{pmatrix}$$

Show that

$$\det(A^T A) = \left(\det \begin{pmatrix} u_1 & v_1 \\ u_2 & v_2 \end{pmatrix}\right)^2 + \left(\det \begin{pmatrix} u_1 & v_1 \\ u_3 & v_3 \end{pmatrix}\right)^2 + \left(\det \begin{pmatrix} u_2 & v_2 \\ u_3 & v_3 \end{pmatrix}\right)^2$$

3. Prove that $\text{Det} \begin{pmatrix} A & B \\ 0 & C \end{pmatrix} = \text{Det}(A)\text{Det}(C)$, where A and C are square matrices (not necessarily of the same size).

4. Let A be an $n \times n$ matrix with rows $a_1, \ldots, a_n$. Compute the determinant of the matrix B with rows $a_n, \ldots a_1$ in terms of the determinant of A.