Due Tuesday, April 5 at 11:59 PM. Please turn in on Gradescope. All chapter and exercise numbers refer to Friedberg, Insel, and Spence's *Linear Algebra*, 5th edition.

Homework policies: You may work together with other students, but solutions must be written on your own without reviewing others' written work. The use of online solution guides, paid resources (e.g. Chegg), and forums (e.g. Math Stack Exchange) is prohibited. If you are struggling with a problem, please review the textbook, consult your classmates, come to office hours, or reach out to the instructors or TAs.

Comments on writing: Your solutions should be written in paragraph form with correct grammar, usage, punctuation, etc. If you introduce new mathematical notation to refer to a certain number, object, equation, etc., be sure to define it clearly. Finally, please write large enough and leave enough blank space so that your paper is easily read. It is much easier for the graders to review and comment on solutions that are spread over three pages than crammed into one.

(1) Let $A, B \in M_{n\times n}(F)$, and suppose that $AB = -BA$. Prove that if n is odd, then A and B cannot both be invertible.

(2) Let $A \in M_{n\times n}(\mathbb{R})$. Prove that $\det(AA^t) \geq 0$. Does this still hold if $A \in M_{n\times n}(\mathbb{C})$? Prove or give a counterexample.

(3) Let E be an elementary $n \times n$ matrix. Prove that $\det(E) = \det(E^t)$.

(4) Prove that the determinant of an upper-triangular $n \times n$ matrix is the product of its diagonal entries.

(5) Compute the determinant of the following matrix by first reducing it to an upper-triangular matrix.

$$A = \begin{pmatrix} 3 & -1 & 5 & 6 \\ 6 & 0 & 8 & 10 \\ 0 & 7 & -5 & -1 \\ -3 & 5 & 0 & 1 \end{pmatrix}$$

(6) You are given an $n \times n$ matrix A with rows $r_1, \ldots, r_n$. Let B be the matrix with rows $r_n, r_{n-1}, \ldots, r_1$. Prove that $\det(B) = (-1)^{\frac{n(n-1)}{2}} \det(A)$.

(7) Suppose that $M \in M_{n\times n}(F)$ can be written as

$$M = \begin{pmatrix} A & B \\ 0 & I \end{pmatrix},$$

where A is a square matrix, and 0 and I are the zero matrix and the identity matrix of appropriate dimensions. Prove that $\det(M) = \det(A)$.

(8) Let $A = (1, -1), B = (2, 5), C = (-3, -7)$ be three points on the plane. Compute the area of the triangle ABC by using determinant.