Due Tuesday, March 15 at 11:59 PM. Please turn in on Gradescope. All chapter and exercise numbers refer to Friedberg, Insel, and Spence's *Linear Algebra*, 5th edition.

Homework policies: You may work together with other students, but solutions must be written on your own without reviewing others' written work. The use of online solution guides, paid resources (e.g. Chegg), and forums (e.g. Math Stack Exchange) is prohibited. If you are struggling with a problem, please review the textbook, consult your classmates, come to office hours, or reach out to the instructors or TAs.

Comments on writing: Your solutions should be written in paragraph form with correct grammar, usage, punctuation, etc. If you introduce new mathematical notation to refer to a certain number, object, equation, etc., be sure to define it clearly. Finally, please write large enough and leave enough blank space so that your paper is easily read. It is much easier for the graders to review and comment on solutions that are spread over three pages than crammed into one.

(1) Let β be the standard ordered basis for $\mathbb{R}^2$. Let α and γ be the ordered bases given by

$$\alpha = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\} \quad \text{and} \quad \gamma = \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right\}.$$

- (a) Find the matrices A and B, where A represents the change of coordinates from α to β and B represents the change of coordinates from γ to β.
- (b) Use the results of part (a) to find matrices representing the change of coordinates from β to α and from β to γ.
- (c) Use your previous work to find matrices representing the change of coordinates from α to γ and from γ to α.

(2) Let α, β, and γ be the bases for $\mathbb{R}^2$ from the previous problem, and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map

$$T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - 2y \\ 3x + y \end{pmatrix}.$$

Compute the following matrix representations of T.

- (a) $[T]_\beta$
- (b) $[T]_\alpha$
- (c) $[T]_\gamma$

(3) Ex. 2.5.7

(4) Ex. 2.5.8. Prove the following generalization of Theorem 2.23.

Theorem. Let V and W be finite-dimensional vector spaces and $T : V \to W$ be linear. Let β and β' be ordered bases for V and γ and γ' be ordered bases for W. Then

$$[T]_{\beta'}^{\gamma'} = P^{-1} [T]_\beta^\gamma Q,$$

where Q is the change of coordinates matrix from β' to β coordinates and P is the change of coordinates matrix from γ' to γ coordinates.

(5) Conider the invertible matrix $A = \begin{pmatrix} 2 & -4 \\ 3 & -5 \end{pmatrix}$. In §3.2 we will see that every invertible matrix can be written as a product of elementary matrices.

- (a) Show that the following sequence of elementary row operations applied to A will result in I_2.
  - (i) Multiply row 1 by 1/2
  - (ii) Add -3 times row 1 to row 2

(iii) Add 2 times row 2 to row 1

(b) Use part (a) to find elementary matrices E_1, E_2, and E_3 such that $E_3E_2E_1A = I_2$.

(c) Use part (b) to write A and A^{-1} as the product of elementary matrices.

(6) Consider the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 5 & 0 \end{pmatrix}.$$

Use the methods of the previous exercise to find A^{-1} by computing the product of elementary matrices. (Other methods for finding the inverse will **not** receive credit.)