Due Tuesday, March 1 at 11:59 PM. Please turn in on Gradescope. All chapter and exercise numbers refer to Friedberg, Insel, and Spence's *Linear Algebra*, 5th edition.

Homework policies: You may work together with other students, but solutions must be written on your own without reviewing others' written work. The use of online solution guides, paid resources (e.g. Chegg), and forums (e.g. Math Stack Exchange) is prohibited. If you are struggling with a problem, please review the textbook, consult your classmates, come to office hours, or reach out to the instructors or TAs.

Comments on writing: Your solutions should be written in paragraph form with correct grammar, usage, punctuation, etc. If you introduce new mathematical notation to refer to a certain number, object, equation, etc., be sure to define it clearly. Finally, please write large enough and leave enough blank space so that your paper is easily read. It is much easier for the graders to review and comment on solutions that are spread over three pages than crammed into one.

(1) Compute the matrix representation $[T]_\beta^\gamma$ for each linear transformation $T : V \to W$, where β and γ are given ordered bases of V and W, respectively.

 (a) $V = \mathbb{R}^3$, $\beta = \{\mathbf{e}_1 - \mathbf{e}_2, \mathbf{e}_2, \mathbf{e}_3 + \mathbf{e}_1\}$, $W = \mathbb{R}^2$, $\gamma = \{2\mathbf{e}_1, -\mathbf{e}_2\}$, and $T(a, b, c) = (a, b + 2c)$.

 (b) $V = M_{2\times 2}(\mathbb{R})$, $\beta = \{2E_{11}, E_{21} - E_{22}, E_{12}, 2E_{22}\}$, $W = \mathbb{R}^3$, $\gamma = \{\mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_1\}$, and

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a, b, c).$$

(2) Let $T : M_{2\times 2}(\mathbb{R}) \to P_3(\mathbb{R})$ be given by

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a + b) + dx + ax^2 + (c - b)x^3.$$

Find ordered bases β and γ for $M_{2\times 2}(\mathbb{R})$ and $P_3(\mathbb{R})$, respectively, such that $[T]_\beta^\gamma$ is the 4×4 identity matrix. Include an argument explaining why your choices of β and γ are indeed bases.

(3) Ex. 2.3.13.

(4) Ex. 2.3.16. (Hint: use the formula $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$ for subspaces W_1 and W_2 of a finite dimensional vector space V.)

(5) Suppose $T : V \to W$ and $U : W \to Z$ are linear transformations.

 (a) Suppose UT is one-to-one. Is T and/or U also one-to-one? For each map, prove or provide a counterexample.

 (b) Suppose UT is onto. Is T and/or U also onto? For each map, prove or provide a counterexample.

 (c) Suppose T and U are bijective (i.e. one-to-one and onto). Show that UT is also bijective.

 (d) Suppose V, W, and Z are finite dimensional and $\dim V = \dim W = \dim Z$. Show that T and U are bijective if UT is bijective.

 (e) Use the map defined in Exercise 2(c) of Problem Set 4 to show that the result in part (d) does not necessarily hold if the finite dimensional condition is removed.

(6) Ex. 2.4.9 (Hint: apply 5(d) to L_A and L_B.)

(7) Ex. 2.4.16