Due Tuesday, February 8 at 11:59 PM. Please turn in on Gradescope. All chapter and exercise numbers refer to Friedberg, Insel, and Spence's *Linear Algebra*, 5th edition.

Homework policies: You may work together with other students, but solutions must be written on your own without reviewing others' written work. The use of online solution guides, paid resources (e.g. Chegg), and forums (e.g. Math Stack Exchange) is prohibited. If you are struggling with a problem, please review the textbook, consult your classmates, come to office hours, or reach out to the instructors or TAs.

Comments on writing: Your solutions should be written in paragraph form with correct grammar, usage, punctuation, etc. If you introduce new mathematical notation to refer to a certain number, object, equation, etc., be sure to define it clearly. Finally, please write large enough and leave enough blank space so that your paper is easily read. It is much easier for the graders to review and comment on solutions that are spread over three pages than crammed into one.

(1) Show that $\beta = \{(1,2,-1),(-2,1,1),(1,2,1)\}$ is a basis of $\mathbb{R}^3$ by finding the unique representation of the vector $(a,b,c) \in \mathbb{R}^3$ as a linear combination of the elements of β.

(2) Find two linearly independent sets S_1 and S_2 in $\mathbb{R}^4$ such that $S_1 \cap S_2 = \varnothing$ and $S_1 \cup S_2$ is linearly dependent.

(3) For each of the following subspaces W, describe an explicit basis β and specify the dimension of W.

  (a) $W \subset \mathbb{R}^n$, where $W = \{(a_1,\dots,a_n) \in \mathbb{R}^n \mid a_1 + \cdots + a_n = 0\}$.
  (b) $W \subset M_{n\times n}(\mathbb{R})$, where W is the space of skew symmetric matrices.
  (c) $W \subset P_n(\mathbb{R})$, where $W = \{p \in P_4(\mathbb{R}) \mid p'(x) = 0 \text{ for all } x \in \mathbb{R}\}$.
  (d) $W \subset P_n(\mathbb{R})$, where $W = \{p \in P_4(\mathbb{R}) \mid p'(0) = 0\}$.

(4) Let W_1 and W_2 be subspaces of $\mathbb{R}^5$ defined as follows:

$$W_1 = \{\mathbf{u} \in \mathbb{R}^5 \mid u_1 + u_3 + u_4 = 2u_1 + 2u_2 + u_5 = 0\} \text{ and } W_2 = \{\mathbf{u} \in \mathbb{R}^5 \mid u_1 + u_5 = 0, u_2 = u_3 = u_4\}.$$

Find a basis of $W_1 \cap W_2$. Then augment this set to a basis of W_1 and a basis of W_2. Explain your reasoning.

(5) Ex. 1.6.22

(6) Ex. 1.6.33