Due Tuesday, February 1 at 11:59 PM. Please turn in on Gradescope. All chapter and exercise numbers refer to Friedberg, Insel, and Spence's *Linear Algebra*, 5th edition.

Homework policies: You may work together with other students, but solutions must be written on your own without reviewing others' written work. The use of online solution guides, paid resources (e.g. Chegg), and forums (e.g. Math Stack Exchange) is prohibited. If you are struggling with a problem, please review the textbook, consult your classmates, come to office hours, or reach out to the instructors or TAs.

Comments on writing: Your solutions should be written in paragraph form with correct grammar, usage, punctuation, etc. If you introduce new mathematical notation to refer to a certain number, object, equation, etc., be sure to define it clearly. Finally, please write large enough and leave enough blank space so that your paper is easily read. It is much easier for the graders to review and comment on solutions that are spread over three pages than crammed into one.

(1) Read p. 22 of the book where the direct sum of two vector spaces is defined. Let W_1 and W_2 be subspaces of a vector space V. Prove that $V = W_1 \oplus W_2$ if and only if each vector in V can be uniquely written as $x_1 + x_2$, where $x_1 \in W_1$ and $x_2 \in W_2$.

(2) A matrix M is called *skew symmetric* if $M^t = -M$. Let U b e the set of all $n \times n$ skew symmetric matrices with entries in $\mathbb{R}$.

   (a) Prove that U is a subspace of $M_n(\mathbb{R})$.

   (b) Let W be the subspace of $M_{n\times n}(\mathbb{R})$ consisting of all symmetric $n \times n$ matrices. Prove that $M_{n\times n}(\mathbb{R}) = U \oplus W$.

(3) For each set S contained in a vector space V, determine whether S spans V. Justify your answers. Your explanation should only use methods covered through section 1.5.

   (a) $V = \mathbb{R}^3$ and
$$S = \{(1,2,-1),(0,1,1),(2,1,-5),(-3,1,0)\}$$

   (b) $V = M_{2\times 2}(\mathbb{R})$ and
$$S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right\}$$

   (c) V is the space of symmetric 2×2 matrices and
$$S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\}$$

(4) Ex. 1.4.16

(5) Ex. 1.5.14