To review for the midterm

- Review your notes, the text, the homework problems, and the suggested exercises in the schedule.
- Do all true false questions in the text. The exam **will** include a significant T/F section.
- Below is a list of topics for the exam and a sampling of questions from past exams. Review these also.
- VERY IMPORTANT. These are just questions from old exams for your **practice**. The questions on our exam may not be similar.

Linear transformations and matrics

- Matrix representations.
- Composition of linear transformations and matrix multiplication
- Isomorphism and Invertibility
- Computing change of coordinate matrices

Elementary Operatations and Systems of linear equations

- Know the rank of a matrix and be comfortable computing inverses using augmented matrices.
- Reduced Row Echelon form and row reduction.
- Column and row operations and multiplication by elementary matrices.
- Definition and properties of homogeneous and inhomogeneous systems and solutions.
- consistent and inconsistent systems.

Example Problems from old exams

(1) Let $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be defined by $T(a + bx + cx^2) = a - 3b + 5cx + (a + c)x^2$.. Let $\gamma = \{1 - x^2, x^2 + x, -5 + 4x^2\}$. Find $[T]_\gamma$. Prove that T is an isomorphism.

(2) We say that A is a submatrix of B if we have

$$B = \begin{pmatrix} * & * & * \\ * & A & * \\ * & * & * \end{pmatrix},$$

where the $''*''$ can be any matrices (of the appropriate dimensions). Prove that

$$\text{rank(A)} \leq \text{rank(B)}.$$

(3) Let

$$A = \begin{pmatrix} 1 & -2 & -1 \\ -2 & 6 & 2 \\ 0 & 1 & 3 \\ 3 & -4 & -2 \end{pmatrix}$$

Solve the linear system $Ax = 0$. Let $b = (1, 0, 4, k)^T$, and consider the system $Ax = b$. Find a real number k such that the system is inconsistent, or explain why this is not possible. Find a real number k such that the system has infinitely-many solutions, or explain why this is not possible. Find a real number k such that the system has exactly one solution, or explain why this is not possible.

(4) Let the reduced row echelon form of A be

$$\begin{pmatrix} 1 & -3 & 0 & 4 & 0 & 5 \\ 0 & 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Determined A if the first, third, and fifth columns of A are

$$\begin{pmatrix} 1 \\ -2 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 2 \\ -4 \end{pmatrix}, \begin{pmatrix} 3 \\ -9 \\ 2 \\ 5 \end{pmatrix}$$

(5) n Let a and b be two distinct real numbers, and consider T from $P_1(\mathbb{R})$ to $\mathbb{R}^2$ defined by $T(f(x)) = (f(a), f(b))$. Show that T is linear and invertible. Given c and d, compute $T^{-1}(c, d)$.

(6) Let V and W be vector spaces. Prove that V and W are isomorphic if and only if there are bases β and γ for V and W respectively and a transformation $T : V \to W$ such that $[T]_\beta^\gamma$ is the identity matrix.

(7) Let $B = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$ and define the mapping $T : M_{2\times 2}(\mathbb{R}) \to \mathbb{R}$ by $T(A) = trace(AB)$. Show that T is linear and compute the rank of T. Show that $\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$ is in the nullspace of T. Find a basis for $N(T)$ which contains this matrix.

(8) in $\mathbb{R}^2$, Let $\beta = \{(1, 2), (3, 4)\}$ and $\beta' = \{(2, 4), (4, 6)\}$. Find the change coordinate matrix taking β' coordinates to β coordinates.

(9) Find all solutions to the system

$$\begin{aligned} x_1 + 2x_2 + 5x_3 &= 1 \\ x_1 - x_2 - x_3 &= 2. \end{aligned}$$

Write down a product of elementary matrices that transforms the matrix of the system to its reduced row echelon form

(10) How many solutions can a homogeneous system of linear equations have? Give an example of a system of two equations in two variables for each case. Explain your examples briefly. You do not have to find the solutions of the systems.