MTH 235 Final, May 8, 2012

NAME (please print): ______________________________

STUDENT ID (please print): ______________________________

PART A	VALUE	SCORE	PART B	VALUE	SCORE
1.	20		6.	30	
2.	20		7.	10	
3.	20		8.	20	
4.	20		9.	20	
5.	20		10.	20	
TOTAL A	100		TOTAL B	100	

Part A.

1. Let $V = \mathbb{R}$ and $W = P_1(\mathbb{R})$. Define $Z = V \times W = \{(v, w) : v \in V \quad \text{and} \quad w \in W\}$. Then Z is a vector space over $\mathbb{R}$ with the operations

$$(v_1, w_1) + (v_2, w_2) = (v_1 + v_2, w_1 + w_2) \quad \text{and} \quad c(v_1, w_1) = (cv_1, cw_1)$$

for $v_1, v_2 \in V$, $w_1, w_2 \in W$ and $c \in \mathbb{R}$.

(a) Find a basis for Z and justify your answer.

(b) Prove that Z is isomorphic to $\mathbb{R}^3$ by defining an invertible linear transformation $T : \mathbb{R}^3 \to Z$.

2. Define $W = \{T \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}) : T(1,0,0) = 0\}$.

(a) Prove that W is a subspace of $\mathcal{L}(\mathbb{R}^3, \mathbb{R})$.

(b) What is the dimension of W? Justify your answer.

3. Let $V = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 + x_3 + x_4 = 0\}$ be a subspace of $\mathbb{R}^4$. Let $\beta = \{v_1, v_2, v_3\}$ and $\gamma = \{w_1, w_2, w_3\}$ be bases for V, where $v_1 = (1, 0, -1, 0)$, $v_2 = (1, 0, 0, -1)$, $v_3 = (0, 1, 0, 0)$, $w_1 = (2, 1, -1, -1)$, $w_2 = (3, 2, -2 - 1)$ and $w_3 = (4, 2, -3, -1)$. Find the change of coordinate matrix that changes γ-coordinates into β-coordinates. (The answer is a 3×3 matrix.)

4. Let

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 2 & 4 \\ 3 & 0 & 6 \end{pmatrix}.$$

Find invertible matrices B and C such that the matrix $D = BAC$ is diagonal.

5. Find all solutions to the following system of equations.

$$x_1 + 2x_2 - x_3 = 3$$

$$2x_1 + x_2 + x_3 = 6$$

Part B.

6. If the statement is true, then write TRUE. Otherwise, write FALSE.

(a) The determinant of an upper triangular matrix equals the product of its diagonal entries.

ANSWER: ____________________________.

(b) Every system of n linear equations in n unknowns can be solved by Cramer's rule.

ANSWER: ____________________________.

(c) Any two eigenvectors are linearly independent.

ANSWER: ____________________________.

(d) If λ_1 and λ_2 are distinct eigenvalues of a linear operator T, then $E_{\lambda_1} \bigcap E_{\lambda_2} = \{0\}$.

ANSWER: ____________________________.

(e) If T is a linear operator on a finite-dimensional vector space V and W is a T-invariant subspace of V, then the characteristic polynomial of T_W divides the characteristic polynomial of T.

ANSWER: ____________________________.

(f) If $< x, y >= 0$ for all x in an inner product space, then $y = 0$.

ANSWER: ____________________________.

7. Prove that if $A, B \in M_{n\times n}(F)$ are similar, then their characteristic polynomials are the same.

8. Let $V = P_2(\mathbb{R})$ and define a linear operator $T : V \to V$ by $T(a + bx + cx^2) = a + b + c + bx + (b + 2c)x^2$. Find a basis β for V such that $[T]_\beta$ is a diagonal matrix.

9. (a) Let V be an inner product space and suppose that x and y are orthogonal vectors in V. Prove that

$$||x+y||^2 = ||x||^2 + ||y||^2.$$

(b) Let $V = P_{2012}(\mathbb{R})$ and define

$$< f(x), g(x) >= \int_0^1 f'(x) g'(x) dx,$$

where $'$ denotes differentiation. Is this an inner product on V? Explain why.

10. Let $V = P_2(\mathbb{R})$ with the inner product $< f(x), g(x) >= \int_0^1 f(x)g(x)dx$. Use the Gram-Schmidt process to replace the standard ordered basis $\beta = \{1, x, x^2\}$ by an orthogonal basis $\{v_1, v_2, v_3\}$ for V.