Math 235: Linear Algebra

Midterm Exam 2

November 20, 2014

NAME (please print legibly): ______________________________
Your University ID Number: ______________________________
Please circle your professor's name: Bobkova Friedmann

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Clearly circle or label your final answers, on those questions for which it is appropriate.**

QUESTION	VALUE	SCORE
1	20	
2	10	
3	15	
4	10	
5	15	
6	10	
7	20	
TOTAL	100	

1. (20 points)

Suppose that V is finite dimensional and let $S : V \to V$ and $T : V \to V$ be linear operators. Prove that ST is invertible if and only if both S and T are invertible.

2. (10 points) In $\mathbb{R}^2$, let $\beta = \{(1,2),(3,4)\}$ and $\beta' = \{(2,4),(4,6)\}$. Find the change of coordinate matrix that changes β' coordinates into β coordinates.

3. (15 points)

(a) Find all solutions for the system

$$\begin{aligned} x_1 + 2x_2 + 5x_3 &= 1 \\ x_1 - x_2 - x_3 &= 2 \end{aligned}$$

(b) Write down a product of elementary matrices which transforms the matrix of coefficients from part a) to its reduced row echelon form.

(c) Find all solutions for the system

$$\begin{aligned} x_1 + 2x_2 + 5x_3 &= 0 \\ x_1 - x_2 - x_3 &= 0 \end{aligned}$$

4. (10 points) Suppose A is a matrix and its row reduced echelon form is

$$\begin{pmatrix} 1 & 2 & 0 & 3 & 8 \\ 0 & 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(a) Compute the dimension of $Null(L_A)$.

(b) If the first column of A is $\begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$ and the third column of A is $\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$, what is A? Show your work.

(c) Find a basis for the vector space spanned by the columns of the matrix A.

5. (15 points) How many solutions can a homogeneous system of linear equations have? Give an example of a system of two equations in two variables for each case. Explain your examples briefly. You do not have to find the solutions of the systems.

6. (10 points) Let

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{pmatrix}.$$

For which $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ does the system $Ax = b$ have a solution?

7. (20 points)

(a) (5 points) Prove that if B is a 2×1 matrix and C is a 1×2 matrix then the 2×2 matrix BC has rank at most one.

(b) (15 points) Show that if A is any 2×2 matrix of rank 1 then there exist a 2×1 matrix B and 1×2 matrix C such that $A = BC$. (You will get 40% credit for providing an example.)