Math 235: Linear Algebra

Midterm Exam 2

November 19, 2013

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Please circle your professor's name: **Friedmann** **Tucker**

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Clearly circle or label your final answers, on those questions for which it is appropriate.**

QUESTION	VALUE	SCORE
1	10	
2	7	
3	18	
4	10	
5	20	
6	20	
7	15	
TOTAL	100	

1. (10 points)

Let $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix}$. Find elementary matrices $E_1, \ldots, E_n$ such that

$$E_n \ldots E_1 A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

2. (7 points) Find the general form for the solutions to the set of linear equations:

$$x_1 + x_2 + 2x_3 = 2$$
$$x_1 + 2x_2 + x_3 = 5$$

3. (18 points) Let $A = \begin{pmatrix} 1 & 3 & 11 & 2 & 12 \\ 2 & 0 & 4 & 0 & 4 \\ 3 & 1 & 9 & 2 & 16 \\ 4 & 2 & 14 & 1 & 13 \end{pmatrix}$. The row reduced form of A is $\begin{pmatrix} 1 & 0 & 2 & 0 & 2 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

(a) Find a basis for the null space of L_A. (Explain your answer.)

(b) Find a basis for the range of L_A. (Explain your answer.)

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(continued)

$$A = \begin{pmatrix} 1 & 3 & 11 & 2 & 12 \\ 2 & 0 & 4 & 0 & 4 \\ 3 & 1 & 9 & 2 & 16 \\ 4 & 2 & 14 & 1 & 13 \end{pmatrix} \text{ The row reduced form of } A \text{ is } \begin{pmatrix} 1 & 0 & 2 & 0 & 2 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(c) Is $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 11 \\ 4 \\ 9 \\ 14 \end{pmatrix} \right\}$ a basis $R(L_A)$? (Explain your answer.)

(d) Is $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 12 \\ 4 \\ 16 \\ 13 \end{pmatrix} \right\}$ a basis $R(L_A)$? (Explain your answer.)

4. (10 points) Suppose B is a matrix and that its row reduced form is $\begin{pmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 2 & 0 \end{pmatrix}$.

Suppose the first column of B is $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and the third column is $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

(a) What is the second column of B?

(b) What is the fourth column of B?

(c) What is the fifth column of B?

5. (20 points)

(a) Let A be an $n \times n$ matrix and let S be the set of all $n \times n$ matrices C such that $CA = 0$. Prove that S is a subspace of $M_{nn}(\mathbb{R})$ (where $M_{nn}(\mathbb{R})$ is the space of all $n \times n$ matrices with coefficients in $\mathbb{R}$).

(b) Let A and B be two $n \times n$ matrices and suppose that $\mathrm{Rank}(AB) = \mathrm{Rank}(B)$. Show that $\mathrm{N}(L_A) \cap \mathrm{R}(L_B) = \{0\}$ (where L_A is the map from $\mathbb{R}^n$ to $\mathbb{R}^n$ sending v to Av and L_B is the map from $\mathbb{R}^n$ to $\mathbb{R}^n$ sending v to Bv, as usual).

6. (20 points) In all of the questions below, $T, U : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ are linear maps. You should explain your answers carefully.

(a) True or false and explain: If UT is onto, then U is onto.

(b) True or false and explain: if U is onto, then UT is onto.

(c) If T is one-one, then $\text{Rank}(T) = n$.

(d) True or false and explain: $\text{Rank}(U + T) \leq \text{Rank}(U)$.

(e) True or false and explain: if U and T are onto, then $\text{Rank}(U + T) = n$.

7. (15 points) In this problem A is an $m \times n$ matrix (m rows and n columns) representing a system of m equations and n unknowns.

(a) Suppose there is a $b \in \mathbb{R}^m$ such that there is exactly one $x \in \mathbb{R}^n$ for which $Ax = b$. Show that the homogenous equation $Ax = 0$ has exactly one solution.

(b) Suppose now that $m > n$. Show that there is a $w \in \mathbb{R}^m$ such that there is no solution to the equation $Av = w$.