Math 235: Linear Algebra

Midterm Exam 1

October 21, 2014

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Please circle your professor's name: Bobkova Friedmann

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Clearly circle or label your final answers, on those questions for which it is appropriate.**

QUESTION	VALUE	SCORE
1	25	
2	10	
3	25	
4	20	
5	10	
6	10	
TOTAL	100	

1. (25 points) Let $T : M_{2\times 2}(R) \to R^2$ be a transformation defined for

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

by

$$T(A) = (a_{11} - a_{21}, -2a_{11} + 3a_{22}).$$

(a) Prove that T is linear.

(b) Find the kernel of T (denoted $Null(T)$ or $N(T)$ or $ker(T)$) and provide a basis for it.

(c) Find the range of T (denoted $Range(T)$ or $R(T)$ or $Im(T)$) and provide a basis for it.

(d) Find the matrix representation $[T]_\alpha^\beta$ where α is the standard basis of $M_{2\times 2}(R)$ and β is the standard basis of R^2.

(e) Show that the rank-nullity theorem holds for T.

2. (10 points) Mark the following statements as True or False. No explanation necessary.

(a) Let V be a vector space. If W_1 and W_2 are both subspaces of V, then $W_1 = W_2$. **True** **False**

(b) The intersection of any two subspaces W_1 and W_2 of a vector space V is a subspace of V. **True** **False**

(c) If W is a subspace of V and Z is a subspace of W, then Z is a subspace of V. **True** **False**

(d) If V is a vector space having dimension n, and if S is a subset of V with n vectors, then S is linearly independent if and only if S spans V. **True** **False**

(e) Every finite dimensional vector space has a unique basis. **True** **False**

3. (25 points) Recall that if A is a 3×3 matrix, $tr(A)$ is defined as the sum of the diagonal entries of A, i.e. $tr(A) = a_{11} + a_{22} + a_{33}$.

Let $W = \{A \in M_{3\times 3}(R) \mid tr(A) = 0\}$ be a subset of the vector space of 3×3 matrices with real entries.

(Note: you may do this problem for 2×2 matrices for 60% partial credit).

(a) Prove W is a subspace of $M_{3\times 3}(R)$.

(b) Find a basis for W.

(c) Find the dimension of W.

4. (20 points) Suppose $(v_1, \ldots, v_n)$ is linearly independent set in vector space V and $w \in V$. Prove that if $(v_1 + w, \ldots v_n + w)$ is linearly dependent, then $w \in \text{Span}(v_1, \ldots v_n)$.

5. (10 points)

(a) Give an example of vector spaces V and W and a linear map $T : V \to W$ such that T is one-to-one but not onto.

(b) Give an example of vector spaces V and W and a linear map $T : V \to W$ such that T is onto but not one-to-one.

6. (10 points) Let $v_1, \ldots v_k, v$ be vectors in a vector space V and define $W_1 = \text{Span}\{v_1, \ldots v_k\}$ and $W_2 = \text{Span}\{v_1, \ldots, v_k, v\}$. Prove that $\dim W_1 = \dim W_2$ if and only if $v \in W_1$.