Math 210, Fall 2021

Problem Set # 8

Due March 30, 2022 at 11:59pm on Gradescope

Question 1. Tesla stock (TSLA) is currently trading at \$1200. You think it is overvalued and decide to short the stock. You're worried, however, that if the stock instead keeps going up, you could owe an arbitrarily large amount of money on this short position.

(a) If you want to keep the short position but cap your possible losses at at most \$300 a year from now, what option(s) should you add to your portfolio?

(b) Suppose instead that your broker will not sell you a short position in the stock, and is only offering (long or short) call options on the stock. Construct a portfolio whose payout is positive if and only if one year from now the stock price has dropped to between \$600 and \$800.

(c) Now suppose your broker is only offering put options. Construct a portfolio with the same payout as that in (b) using only puts.

Question 2. a) Draw a payout graph for the following two call spread portfolios:

(i) long one K-call and short one $(K+1)$-call.

(ii) long two K-calls and short two $(K+.5)$-calls.

b) A digital call with strike price K and maturity T pays out 1 if $S_T \geq K$ and 0 if $S_T < K$. By constructing a series of portfolios of call spreads and taking limits, prove that the price at time t of a digital all, with strike K^* and payout 1, is given by

$$-\frac{\partial}{\partial K} C_K(t,T)\big|_{K^*},$$

where $\big|_{K^*}$ means the function is evaluated at $K = K^*$. *Hint: recall the limit definition of the derivative from Calc I.*

c) A digital put with strike price K and maturity T pays out 1 if $S_T \leq K$ and 0 if $S_T > K$. Write down the equivalent formula for a digital put option in terms of put prices.

d) By examining the payout profile, derive a put-call parity relationship for the digital call and digital put.

Question 3. We showed in class that the price at time $t \leq T$ of a K call on a stock paying no dividends satisfies

$$C_K(t,T) \geq \max(S_t - KZ(t,T), 0).$$

a) Use the above bound to prove that if $t \leq T_1 \leq T_2$,

$$C_K(t, T_2) \geq C_K(t, T_1).$$

b) Does the same hold for puts? That is, prove or find a counterexample to the statement

$$P_K(t, T_2) \geq P_K(t, T_1) \text{ for } t \leq T_1 \leq T_2.$$

Question 4. An asymmetric call butterfly with strikes (K_1, K_2, K_3) is a portfolio consisting of long 1 K_1 call, short 2 K_2 calls, and long 1 K_3 call, all with maturity T. Find the payout function at maturity T for a call butterfly with strikes $(70, 105, 110)$. Graph the payout at maturity as a function of the stock price.