Math 210, Spring 2022

Problem Set # 6

Due **Friday**, March 4, 2022 at 11:59pm on Gradescope

Question 1. Suppose the current one-year euro swap rate $y_0[0,1]$ is 1.74%, and the two-year and three-year swap rates are 2.24% and 2.55% respectively. Euro swap rates are quoted with annual payments and 30/360 daycount (thus $\alpha = 1$).

a) Use bootstrapping to calculate $Z(0,1), Z(0,2)$ and $Z(0,3)$ and obtain $P_0[0,3]$ the present value of a three-year annuity paying €1 per year.

b) Recall that the annually compounded zero rate for maturity T is the rate r such that $Z(0,T) = (1+r)^{-T}$. Calculate the one-year, two-year and three-year zero rates and compare them to the swap rates. For upward sloping yield curves, that is, when $y_0[0,T_2] > y_0[0,T_1]$ for $T_2 > T_1$, will zero rates be higher or lower than swap rates?

c) An approximate short-cut sometimes used on trading desks to calculate the present value of, say, a three-year annuity is to discount each payment at the three-year swap rate. In other words, it is assumed that $Z(0,n) = (1 + y_0[0,3])^{-n}$ for $n = 1,2,3$. this is often called IRR (internal rate of return) discounting. Calculate the error in valuing the annuity in (a) this way.

d) Calculate the price of a three-year fixed rate bond of notional €1 and annual coupons of 2.55% using the ZCB prices calculated in (a), and verify this equals the price obtained via IRR discounting at a rate of 2.55%

Question 2. We have assumed that the payment dates for the fixed and floating legs of a swap are the same. However, in practice, the payment frequencies may differ. For example, in the US swap market, the fixed leg usually has semi-annual payments ($\alpha = 0.5$) and the floating leg has quarterly payments ($\alpha = 0.25$).

a) Draw a diagram similar to Figure 4.1 on page 34 of Blyth (also in the 10/04 lecture notes) for a swap where the fixed payments are semi-annual and floating payments are quarterly.

b) Consider a swap from T_0 to T_n with fixed rate K (assume n is the number of fixed payments). Suppose the term length for the floating leg is $\alpha(\text{FL})$ and the term length for the fixed leg is $\alpha(\text{FXD})$. Write down a formula for the value of the swap $V_K^{SW}(t)$ (where $t \leq T_0$) in terms of ZCB prices.

c) This continues (b). Does the value of the swap depend on $\alpha(\text{FL})$? Does it depend on $\alpha(\text{FXD})$?

d) This continues (b). As usual, the swap rate $y_t[T_0, T_n]$ is the special fixed rate such that $V^{SW}_{y_t[T_0,T_n]}(t) = 0$. Is the swap rate larger if the fixed leg payments are quarterly or if they are annually?