

Math 210, Spring 2022

Problem Set # 4

Due February 16, 2022 at 11:59pm on Gradescope

Question 1. Let S_t be the current price of a stock that pays no dividends.

- a) Let r_{bid} be the interest rate at which one can invest/lend money, and r_{off} be the interest rate at which one can borrow money, $r_{\text{bid}} \leq r_{\text{off}}$. Both rates are continuously compounded. Using arbitrage arguments, find upper and lower bounds for the forward price of the stock for a forward contract with maturity $T > t$.
- b) How does your answer change if the stock itself has bid price $S_{t,\text{bid}}$ and offer price $S_{t,\text{off}}$?

Question 2. FX forwards are among the most liquid derivative contracts in the world and often reveal more about the health of money markets (markets for borrowing or lending cash) than published short-term interest rates themselves.

- a) On Oct. 3, 2008 the euro dollar FX rate was trading at $\text{€}1 = \$1.3772$, and the forward price for a maturity April 3, 2009 forward contract was $\$1.3891$. Assuming six-month euro interest rates were 5.415%, what is the implied six-month dollar rate? Both interest rates are quoted with act/360 daycount and semi-annual compounding. There are 182 days between Oct. 3, 2008 and April 3, 2009.
- b) Published six-month dollar rates were actually 4.13125%. What arbitrage opportunity existed? What transactions does a potential arbitrageur need to undertake to exploit this opportunity?
- c) During the financial crisis, several European commercial banks badly needed to borrow dollar cash, but their only source of funds was euro cash from the European Central Bank (ECB). These banks would: borrow euro cash for six months from the ECB; sell euros/buy dollars in the spot FX market; and sell dollars/buy euros six months forward (to neutralize the FX risk on their euro liability). Explain briefly how these actions may have created the arbitrage opportunity in (b), which existed for several months in late 2008.

Question 3. Suppose a stock is currently (time $t = 0$) worth 100. Further, suppose the one year annually compounded interest rate is 2%, and the two year annually compounded rate is 3%. Find the following:

- a) The forward price for a forward contract on the stock with maturity year $T_1 = 1$.

- b) The forward price for a forward contract on the stock with maturity year $T_2 = 2$.
- c) The forward price for a forward contract with maturity $T_1 = 1$ on a ZCB with maturity $T_2 = 2$.
- d) The forward price for a forward contract with maturity $T_1 = 1$ on a forward contract on the stock with maturity $T_2 = 2$ and delivery price $K = 101$.