Math 210, Spring 2022

Problem Set # 2

Due Feb 2, 2022 at 11:59pm on Gradescope

Question 1. The *conditional probability* that event A occurs given that B occurs is denoted $P(A|B)$. A standard, very useful theorem is Bayes' Theorem, which says:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}.$$

A new unusual disease is spreading. Currently one in every one million people has it. A company develops a test for the disease which they say is 99% accurate, in the sense that a person with the disease will test positive 99% of the time, and a person who doesn't have it will test negative 99% of the time.

a) Write what it means for the test to be 99% accurate as two conditional probabilities. Use the events *test positive, test negative, sick, not sick*.

b) If you test positive, what are the odds you have the disease? *Hint: first write this as a conditional probability, then use Bayes' Theorem.*

Question 2. Suppose, instead of a standard annuity, you have the following cashflow:

- A payment of 100 at the end of every first quarter, and

- A payment of 300 at the end of every year.

This lasts for 5 years, i.e. 10 total payments. What is the present value of this cashflow at the start of the first year if the quarterly compounded annual interest rate is 3%? You may assume a 30/360 daycount.

Question 3. Two market standards for US dollar interest rates are semi-annual compounding with 30/360 daycount (semi-bond, denoted y_{sb}) and annual compounding with act/360 daycount (annual-money, denoted y_{am}). *Note: it's important to know how to do this math, but you won't need to know these industry terms on exams*

a) Derive an expression for y_{am} in terms of y_{sb}. You may assume all years have 365 days.

b) Calculate y_{am} when $y_{sb} = 5\%, 6\%$, and 7%.

Question 4. Suppose the continuously compounded annual interest rate is 4%. Find

a) The equivalent annually compounded rate with act/360 daycount.

b) The equivalent semiannually compounded rate with act/365 daycount. Assume quarters have lengths 92, 92, 91, 90 days.

Question 5. A *simple interest rate* of r for T years means a 100 investment becomes $100(1 + rT)$ at maturity T.

a) For simple interest 5% for ten years, calculate the equivalent compound interest rate with (i) annual, (ii) quarterly and (iii) continuous compounding. Assume 30/360 daycount.

b) Show that if simple interest of r for T years is equivalent to r^* interest rate with annual compounding, then $r^* \to 0$ as $T \to \infty$.