Math 210, Spring 2022

Problem Set # 12

Due April 27, 2021 at 11:59pm on Gradescope

Question 1. For each of the following positions I-IX in the table below, determine whether Bank A has vega≥ 0, vega≤ 0, no volatility exposure "=0", or indeterminate volatility exposure "?". Similarly, determine Bank A's delta with respect to the stock price. All options are European with maturity T on a stock that pays no dividends, and $0 < K_1 < K_2$. Justify your answers briefly. Note: "knocking out" means the payout becomes zero when the condition is met.

	Bank A position	(i) Vega $\geq 0, \leq 0, = 0, ?$	(ii) Delta $\geq 0, \leq 0, = 0, ?$
I	long a K_1 call		
II	short a K_1 put		
III	short a K_2, K_1 put spread		
IV	long a $K_1, (K_1+K_2)/2, K_2$ call butterfly		
V	long a K_1 put knocking out if $S_T < K_2$		
VI	long a K_2 put knocking out if $S_T > K_1$		
VII	short a K_1 call and long a K_1 put		
VIII	long a K_1 straddle		
IX	short a forward contract on the stock with delivery price K_1		
X	short a futures contract on the stock		

Question 2. A *capped power call* is a K_1-strike power call whose payout is capped at $K_2 > K_1$. That is, it has payout at time T

$$\begin{cases} 0 & S_T < K_1 \\ (S_T - K_1)^2 & K_1 \leq S_T \leq K_2 \\ (K_2 - K_1)^2 & S_T > K_2 \end{cases}$$

a) Write down (but do not solve) the integral expression for the price of the capped power call.

b) Suppose $K_1 = 100$ and $K_2 = 120$, and that you observe (regular) call option prices $C_{100}(t,T) = 20$ and $C_{120}(t,T) = 10$. By comparing the payout functions of the capped power call with some combination of regular calls, find the least expensive upper bound for the price of the capped power call.

c) Suppose you additionally observe that $C_{110}(t,T) = 16$. Find a greatest lower bound for the price of the capped power call.