Math 210, Spring 2022

Problem Set # 1

Due January 26, 2022 at 11:59pm on Gradescope

Question 1. A fair coin is flipped 7 times. Let X = total number of heads. The sample space consists of all possible sequences of outcomes of the 7 flips.

a) Write down three possible outcomes ω from the sample space Ω.

b) How many outcomes are in the sample space?

c) Compute $P(\{\omega\})$ for each outcome ω you wrote down in part (a).

d) Compute $X(\omega)$ for each outcome ω you wrote down in part (a).

e) Find $P(X \leq 5)$. Hint: Find $P(X > 5)$ first.

Question 2. Consider a coin where the probability of heads is $0 < p < 1$. Do **not** assume $p = 1/2$. Flip it until the first tails occurs. Let X = number of flips needed to see the first tails.

a) How many possible outcomes are in the sample space?

b) Find $P(X = 1)$, $P(X = 2)$, and $P(X = 3)$.

c) Write down a formula for $P(X = k)$, where k is a positive integer.

Question 3. Recall that a *Forward Contract* with maturity T and delivery price K is an agreement that the *long party* will buy one share of stock from the *short party* at time T for price K.

For this problem, suppose $T = 1$ and $K = 110$

a) Suppose your portfolio is long 5 forward contracts. If the stock price at time $T = 1$ is $S_1 = 120$, what is the value of your portfolio at time $T = 1$?

b) Suppose your portfolio is short 5 forward contracts. Write the value at time $T = 1$ as a function of the stock price S_1.

c) Suppose your portfolio is short 5 forward contracts and long 5 shares of the stock. Write the value at time $T = 1$ as a function of the stock price S_1.