Math 210: Midterm Formula Sheet

Forward prices:

Underlying	Forward price
Asset paying no income	$S_t e^{r(T-t)}$
Asset paying known income I	$(S_t - I)e^{r(T-t)}$
Asset paying known dividends at rate q	$S_t e^{(r-q)(T-t)}$

Value of a forward contract:

$$V_K(t,T) = (F(t,T) - K)e^{-r(T-t)}.$$

Forward rate agreement:

The buyer of a FRA with maturity T and fixed rate K agrees at $t \le T$ to pay αK and receive $\alpha L_T[T, T+\alpha]$ at time $T+\alpha$.

Forward libor rate: For $t \le T$ we have

$$L_t[T, T+\alpha] = \frac{Z(t,T) - Z(t, T+\alpha)}{\alpha Z(t, T+\alpha)}.$$

Swap:

Consider a (vanilla) swap with start date T_0, maturity T_n and payment dates $T_1, \dots T_n$ where $T_{i+1} = T_i + \alpha$. The floating leg consists of payments $\alpha L_{T_i}[T_i, T_i + \alpha]$ at $T_i + \alpha$. The fixed leg consists of payments αK at $T_i + \alpha$.

Value of a swap:

At time $t \le T_0$ we have

$$V_K^{\mathrm{SW}}(t) = V^{\mathrm{FL}}(t) - V_K^{\mathrm{FXD}}(t) = (y_t[T_0, T_n] - K)P_t[T_0, T_n]$$

where

$$y_t[T_0, T_n] = \frac{Z(t, T_0) - Z(t, T_n)}{P_t[T_0, T_n]}.$$

Futures contract:

At time t we can go long a futures contract with price $\Phi(t,T)$ at no cost. Each day we receive (pay if negative) the mark-to-market change $\Phi(i,T) - \Phi(i-1,T)$.

Summation Formulas

If $|x| < 1$ **then the following hold:**

$$1 + x + x^2 + \cdots + x^{n-1} = \sum_{k=0}^{n-1} x^k = \frac{x^n - 1}{x - 1}$$

$$1 + x + x^2 + \cdots = \sum_{k=0}^{\infty} x^k = \frac{1}{1 - x}$$

$$x + x^2 + \cdots + x^{n-1} = \sum_{k=1}^{n-1} x^k = \frac{x^n - x}{x - 1}$$

$$x + x^2 + \cdots = \sum_{k=1}^{\infty} x^k = \frac{x}{1 - x}$$

If $r > 0$ **then the following hold:**

$$\frac{1}{1+r} + \frac{1}{(1+r)^2} + \cdots + \frac{1}{(1+r)^n} = \sum_{k=1}^{n} \frac{1}{(1+r)^k} = \frac{1}{r}\left(1 - \frac{1}{(1+r)^n}\right) = \frac{1 - (1+r)^{-n}}{r}$$

$$\frac{1}{1+r} + \frac{1}{(1+r)^2} + \cdots = \sum_{k=1}^{\infty} \frac{1}{(1+r)^k} = \frac{1}{r}$$

$$\frac{1}{1+r} + \frac{2}{(1+r)^2} + \frac{3}{(1+r)^3} + \cdots = \sum_{k=1}^{\infty} \frac{k}{(1+r)^k} = \frac{1+r}{r^2}$$

Trick:

$$\begin{aligned}(1 + x + x^2 + \cdots + x^{n-1})(1 - x) &= 1 - x + x - x^2 + x^2 - x^3 + \cdots + x^{n-1} - x^n \\ &= 1 - x^n\end{aligned}$$

$$\frac{1 - x^n}{1 - x} = 1 + x + x^2 + \cdots x^{n-1}$$