MATH 210

Midterm

October 26, 2017

Last Name (Family Name): ______________________________

First Name: ______________________________

Student ID Number: ______________________________

Circle your Instructor's Name:

Mueller **Hambrook**

Please read the following instructions very carefully:

- You have **75 minutes** to complete this exam.
- Write in pencil or pen.
- No notes, textbooks, calculators, phones, or other electronic devices.
- Show your work and justify your answers. If you need extra space, use the back of the previous page and clearly indicate that you have done so. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. Clearly circle or label your final answers.
- Copy and sign your name to the following academic honesty statement: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

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Signature: ______________________________

QUESTION	VALUE	SCORE
1	16	
2	16	
3	16	
4	16	
5	20	
6	16	
TOTAL	100	

1. (16 points)

Assume there is a fixed interest rate r which is continuously compounded, and you receive the following cash stream. The current time is January 1, 2018. At the end of each month beginning now and ending 2 years from now, you receive a payment which varies according to the month. In months $1, 3, 5, \ldots$ (odd months) you receive \$2, and in months $2, 4, \ldots$ (even months) you receive nothing. Find the present value of the cash stream. Please express your answer in closed form, not just as a sum of a large number of terms.

Hint: Recall the formula

$$1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x},$$

valid for $x \neq 1$.

2. (16 points)

Assume that $t < T_1 < T_2$. Suppose that at time t you enter into a long forward contract on a long forward contract. That is, at the current time t you agree to pay K_1 at maturity date T_1 to enter into a second forward contract. The second forward contract has maturity date T_2, and at that time you have to pay K_2 for a share of a certain stock. The price of the stock is S_t at time t. Assume that there is a fixed continuously compounded interest rate r. Also assume that the forward contract on the forward contract costs you nothing at time t.

Express K_1 as a function of r, t, T_1, T_2, K_2.

Hint: Recall that the value of a forward contract with maturity T and delivery price K on an asset with price X_t at time t is

$$V_K(t,T) = X_t - Ke^{-r(T-t)}$$

and the forward price is

$$F(t,T) = X_t e^{r(T-t)}.$$

Further Hint: Consider the second forward contract as an asset and compute its value at time t.

3. (16 points)

A bank has borrowing needs of N dollars at time $T > 0$. Show that by combining an FRA (forward rate agreement) trade today with a LIBOR loan at time T, the bank can today lock in its interest cost for the period T to $T + \alpha$. Does the borrowing bank need to buy or sell the FRA to do this? What is the fixed rate that the bank locks in?

Hint: Recall that in an FRA, the long counterparty receives amount $\alpha L_T[T, T + \alpha]$ at time $T + \alpha$ and also must pay αK at time $T + \alpha$. Also recall that the forward LIBOR rate is

$$K = L_t[T, T + \alpha] = \frac{Z(t, T) - Z(t, T + \alpha)}{\alpha Z(t, T + \alpha)}.$$

4. (16 points)

At current time t, a certain stock paying no income has price 8 and the forward price with maturity T on the stock is 8. The annually compounded zero rate for period t to T is 3%. Determine whether there is an arbitrage opportunity. If there is, find an arbitrage portfolio. Verify the portfolio you construct is an arbitrage portfolio.

Hint: Recall that, if there is no-arbitrage, then the forward price $F(t, T)$, stock price S_t, and annually compounded zero rate r_A for period t to T are related by

$$F(t, T) = S_t(1 + r_A)^{T-t}.$$

5. (20 points)

Recall that (assuming no-arbitrage) the forward swap rate at $t \leq T_0$ for a swap from T_0 to T_n is given by

$$y_t[T_0, T_n] = \frac{Z(t,T_0) - Z(t,T_n)}{\sum_{i=1}^{n} \alpha Z(t,T_i)} = \frac{\sum_{i=1}^{n} \alpha L_t[T_{i-1}, T_i] Z(t,T_i)}{\sum_{i=1}^{n} \alpha Z(t,T_i)},$$

where $\alpha = T_i - T_{i-1}$ is the time between payments.

Assume we have the following zero coupon bond prices and forward libor rates:

- $Z(0,4) = 0.6$
- $Z(0,5) = 0.4$
- $L_0[3,4] = 0.1$
- $L_0[4,5] = 0.2$

Compute the following. (Since calculators are **not** allowed, you do **not** have to simplify your answers to single numbers. Answers in forms like $\frac{0.2+0.1}{(0.1)(0.6)}$ are acceptable.)

(a) (6 points) If $\alpha = 1$, find $y_0[3,5]$. **Hint:** Use the second version of the formula for $y_t[T_0, T_n]$.

(b) (7 points) Find $Z(0,3)$. **Hint:** First find a formula relating $L_0[3,4]$, $Z(0,4)$, and $Z(0,3)$.

(c) (7 points) If $\alpha = 1$ and if the forward swap rate $y_0[3,6]$ is 0.2, find $Z(0,6)$. **Hint:** Use the first version of the formula for $y_t[T_0, T_n]$ and your answer from part (b).

6. (16 points)

Let $t \leq T \leq T + \alpha$. Let S_t denote the price of a stock at time t. Consider a derivative contract where the long counterparty agrees at t to receive 1 share of stock at time $T + \alpha$ and pay $\alpha L_T[T, T + \alpha]$ cash at time $T + \alpha$. The short counterparty agrees to the opposite. Use a replication argument to find the value of this derivative to the long counterparty at time t.

Hint: Recall that the value at t of a forward contract on the stock with maturity $T + \alpha$ and delivery price M is

$$V_M^{\text{FOR}}(t, T + \alpha) = S_t - MZ(t, T + \alpha)$$

and the value at t of a FRA with maturity T, term length α, and fixed rate K is

$$V_K^{\text{FRA}}(t, T) = Z(t, T) - Z(t, T + \alpha) - \alpha K Z(t, T + \alpha).$$