MTH 210 **Midterm** **October 27, 2016**

Last/Family Name: _____

First/Given Name: _____

Student ID Number: _____

Instructor (circle): Hambrook (MWF 10:25) Zhong (MW 12:30)

Honor Pledge: "I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own."

You must write out and sign the honor pledge for your examination to be valid.

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Signature: _____ Date: _____

Instructions:

- Time: 75 minutes.
- Write in pencil or pen.
- If you need extra space, use the back of the page, and indicate it.
- You are allowed a one page (front and back, standard size) sheet of notes.
- You are allowed a non-programmable, non-graphing calculator.
- No other notes, textbooks, phones, or other electronic devices.
- To receive full credit, you must show your work and justify your answers.

QUESTION	VALUE	SCORE
1	3	
2	2	
3	3	
4	3	
5	3	
6	3	
TOTAL	17	

1. (3 points) Give complete definitions of the following:

(a) Zero coupon bond.

(b) Forward contract.

2. (2 points) Use the replication theorem to show that if the interest rate with compounding frequency m for period t to T has value r, then

$$Z(t,T) = (1 + r/m)^{-m(T-t)}.$$

3. (3 points) The one-year, two-year, and three-year continuous zero rates are 3%, 6%, and 9% respectively.

(a) Find the two-year libor rate.

(b) Find the the one-year forward two-year libor rate.

(c) If the two-year forward two-year libor rate is 16%, find the four-year continuous zero rate.

4. (3 points) Use a replication argument to prove that the value at current time t of a FRA (forward rate agreement) with maturity T, fixed rate K, and term length α is

$$V_K(t,T) = Z(t,T) - Z(t,T+\alpha) - \alpha K Z(t,T+\alpha).$$

5. (3 points) Assume the current time is $t = 0$. Consider a swap starting now ($T_0 = t = 0$) and ending in two years ($T_n = 2$) with fixed rate 3% and quarterly payments. Assume the quarterly compounded zero rates for all payment times are 2%. Find the present value of

(a) the floating leg

(b) the fixed leg

(c) the swap

6. (3 points) Suppose a stock pays dividends m times per year at evenly spaced times with annual yield q. So each dividend payment is equal to q/m of the stock price and the payments are made at times $t+1/m$, $t+2/m$, ..., where t is the current time. Suppose the dividends are automatically reinvested in the stock.

Result. If $T-t$ is an integer multiple of $1/m$ (that is, $T-t=n/m$ for some integer n), then the forward price for the stock is

$$F(t,T)=\frac{S_t(1+q/m)^{-m(T-t)}}{Z(t,T)}. \qquad (*)$$

Do **NOT** prove this result. It is stated for context.

Suppose $m=5$ and $T-t=1/2$ (so $T-t$ is not an integer multiple of $1/m$). Show that if $(*)$ holds, then you can build an arbitrage portfolio. Verify the portfolio is an arbitrage portfolio.