Formula sheet

Geometric Series Formula

$$\sum_{i=1}^{n} r^i = \frac{r(1-r^n)}{1-r} \qquad \sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r}$$

Forward: Value and Forward Price

$V_K(t,T) = (F(t,T)-K)Z(t,T)$

$$F(t,T) = \begin{cases} S_t/Z(t,T) & \text{asset paying no income} \\ (S_t - I)/Z(t,T) & \text{asset paying known income of present value } I \\ S_t e^{-q(T-t)}/Z(t,T) & \text{asset paying dividends at continuous rate } q \\ X_t e^{(r_d - r_f)(T-t)} & \text{foreign currency } (r_d,\ r_f\text{: domestic, foreign continuous rates}) \end{cases}$$

FRA: Value and Forward Libor Rate

$$V_K(t,T) = Z(t,T) - Z(t,T+\alpha) - \alpha K Z(t,T+\alpha), \qquad L_t[T,T+\alpha] = \frac{Z(t,T)-Z(t,T+\alpha)}{\alpha Z(t,T+\alpha)}.$$

Swap: Value and Forward Swap Rate

$$V_K^{\text{SW}}(t) = V^{\text{FL}}(t) - V_K^{FIX}(t) = (Z(t,T_0) - Z(t,T_n)) - \alpha K \sum_{i=1}^{n} Z(t,T_i),$$

$$y_t[T_0,T_n] = \frac{Z(t,T_0)-Z(t,T_n)}{\alpha \sum_{i=1}^{n} Z(t,T_i)}$$

The Black-Scholes Model

$$\log S_T | S_t \sim N\left(\nu, \sigma^2(T-t)\right), \text{ where } \nu = \log S_t + \left(r - \frac{1}{2}\sigma^2\right)(T-t).$$

$$\mathbb{E}_*\left[g(S_T)|S_t\right] = Z(t,T) \int_{-\infty}^{\infty} \frac{g(e^y)}{\sqrt{2\pi\sigma^2(T-t)}} e^{-(y-\nu)^2/(2\sigma^2(T-t))} dy.$$

$$C_K(t,T) = S_t \Phi(d_1) - K Z(t,T) \Phi(d_2),$$

where

$$d_1 = \frac{\log(S_t/K) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \qquad \text{and } d_2 = d_1 - \sigma\sqrt{T-t}.$$

$$\text{Delta } = \frac{\partial C_K(t,T)}{\partial S_t} = \Phi(d_1), \qquad \text{Vega } = \frac{\partial C_K(t,T)}{\partial \sigma} = \frac{S_t\sqrt{T-t}}{\sqrt{2\pi}} e^{-d_1^2/2}$$