MATH 209: Homework 8 – due Friday, 3/29 at 2pm on gradescope

(P1) Find L, W, L_q, W_q for the $M/M/\infty$ queue.

(P2) Derive the formulas for p_n for all n, for the finite source $M/M/c$ queue given in remark 4.4.1 in the chapter 4 notes.

(P3) This problem deals with "impatient customers". Suppose customers arrive to a store according to a Poisson process with intensity λ and service times are i.i.d. exponential variables with mean $1/\mu$. A queue of unrestricted size is allowed to form. Answer the following.

(a) Suppose there is a single server and if a customer arrives when the server is busy, they enter the queue with probability $a > 0$ and balk with probability $1 - a > 0$. Solve for the steady-state probabilities p_n for the number of customers in the system at an arbitrary time. When does a steady-state exist? Note: sum all infinite series in this part.

(b) Again there is a single server, but now all customers enter the system regardless of the number of customers in the system when they arrive (no balking). Suppose instead, after they have been waiting for awhile, customers in queue will decide to leave the queue and exit the system. In particular, assume the time until a customer leaves the queue and exits the system is exponentially distributed with mean $1/\alpha$. Solve for the steady-state probabilities p_n for the number of customers in the system at an arbitrary time in terms of p_0, and write p_0 in terms of a series (do not try to sum the series).

(c) Extra credit: When does a steady-state exist in part (b)? Prove your answer.

(P4) Consider the system in P3(a), where customers balk with probability $1 - a$.

(a) Solve for p_n if there are 2 servers.

(b) Find formulas for p_n for all n in the general case of c servers.