## MATH 209: Homework 5 - due Friday, 2/23 at 2pm on gradescope

## Notes:

- For this assignment, you will want to use a calculator that can compute matrix operations and find eigenvalues/eigenvectors.
- Throughout, you may assume that all recurrent states are also strongly recurrent.
(P1) Abigail spends her entire weekly allowance on either candy or toys. If she buys candy one week, she is 60 percent sure to buy toys the next week. The probability that she buys toys on two successive weeks in $\frac{1}{5}$.
(a) Set up this process as a Markov chain. Determine the transition matrix $P$ and draw a state diagram.
(b) If Abigail bought a toy this week. Find the probability that she will buy candy two weeks from now.
(c) Explain how you know the limiting theorem applies and find the limiting distribution $\langle\pi|$. Note: do this by hand.
(d) What fraction of weeks does Abigail spend her allowance on candy?
(P2) In studying the vegetation state of a certain habitat, we can divide it into 5 states: barren, grassland, brushland, young forest and mature forest. Assume after studying years of data in the state transitions of various habitats in a region, that it has been observed that the transitions are reasonably modeled by a Markov chain with probabilities given as follows: (timestep=one decade)
- Currently barren: Still barren after a decade with probability 0.8 , developed into a grassland with probability 0.2 .
- Currently grassland: Still grassland after a decade with probability 0.7, developed into a brushland with probability 0.2 , regressed to barren with probability 0.1 .
- Currently brushland: Still brushland after a decade with probability 0.6, regressed to grassland with probability 0.2 , regressed to barren with probability 0.1 and developed into young forest with probability 0.1.
- Currently young forest: Still young forest with probability 0.8 , developed into mature forest with probability 0.1 , regressed to either grassland or brushland with equal probability (due to catastrophe) but no probability to transition to barren state.
- Currently mature forest: Stays a mature forest with probability 0.92 but possible regression to any of the 4 previous states with equal probability.
(a) Write down the transition matrix for the underlying Markov Chain. Label the states corresponding to each row/column clearly.
(b) If a habitat is currently in the grassland state, what is the chance that it is a young forest 3 decades from now?
(c) Explain how you know the limiting theorem applies and find the limiting distribution $\langle\pi|$.
(d) In the long-run, which habitats are most likely and least likely, and what fraction of habitats will be in each state?
(P3) This questions refers to the Markov chain in HW4 P6.
(a) Is this chain irreducible? Write down the communication classes.
(b) Which states are transient, which are recurrent? Are there any absorbing states? In the long-run, which states can we expect to find the system in?

