MATH 209: Homework 4 – due Friday, 2/16 at 2pm on gradescope

(P1) A distributor of seasonal items wants to establish the best stock level to purchase. The items will sell for \$25.00 and will cost the distributor \$10.00. Units unsold at the end of the season will be sold to a third party for \$5.00 per unit. Assume the demand for an item has mean 100 units and a standard deviation of 10 units.

(a) If demand for an item during the season is normally distributed, how many units should be purchased to maximize expected profit?

(b) If demand for an item during the season is uniformly distributed, how many units should be purchased to maximize expected profit?

(P2) The NFL has 32 teams. Suppose each team has exactly 67 active players on their roster during the season. There is a draft each year where 7 new players are added to each team. Since the total for each team must be 67, teams are forced to cut (remove) existing players to make room for new ones.

(a) Assuming a player can only join a team by being selected in the draft, estimate the average career length of a football player in the NFL.

(b) Now suppose players that were not lucky enough to be drafted may enter the league by signing directly with a team, if the team desires. Suppose the average career length in the NFL is 3.5 years. With the same assumptions as described above, estimate the average number of players who enter the league each year without being drafted.

(P3) *Trash Plate Heaven* does take-out orders only. During peak periods two servers are on duty and management estimates they are only idle 1 percent of the time. Management would like to increase that number to allow for breaks.

(a) If a third server is added, estimate the idle time for each server?

(b) Suppose after adding a 3rd server the pressure on each server is reduced so they can work more carefully. However, the service output rate (customers served per time) of each server is reduced by 20%. Estimate the percent idle time for each server.

(c) Suppose management hires an aid instead of a 3rd server. The aid is paid a lower salary and does simple tasks to lessen the load on the 2 servers. This allows the 2 servers to decrease their average service time (time per customer served) by 20% (note: this is relative to the original service rate, ignoring part (b)). Estimate the percent idle time of the 2 servers.

(P4) A store has m independent tellers with identical exponentially distributed service times. At closing time all tellers are busy, and there are n more customers are waiting. Although no more customers may enter the store, the workers cannot leave until all customers are served. Find the average time for the system to empty completely.

(P5) A political scientist in Canada discovered that of the children of Conservative parents, 80 percent vote Conservative and the rest vote Labor, of the children of Labor supporters, 60 percent vote Labor, 20 percent vote Conservative and 20 percent vote for the New Democratic Party (NDP), and of the children of NDP supporters, 75 percent vote NDP, 15 percent vote Labor and 10 percent vote Conservative. (for the purposes of this question assume that both parents always have the same political preference)

(a) Set up this process as a Markov chain with steps corresponding to successive generations. Determine the transition matrix and sketch the state diagram.

(b) What is the probability that the grandchild of a Conservative will vote for the NDP?

(c) Suppose at time step 0 a randomly chosen person from the Canadian population has a 40 percent chance of voting Conservative, a 50 percent chance of voting Labor and otherwise will vote NDP. Write down the current state distribution $\langle w^{(0)}|$. Find the state vector $\langle w^{(2)}|$ describing the situation 2 generations from now.

(P6) A trio of nineteenth century Russian noblemen fight a three-way duel. The three men A, B and C are of different abilities in pistol shooting. They have respective probabilities of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$ of hitting and killing the target at which they aim. In each round of the duel, the men shoot simultaneously and each one aims at the best other marksman not yet killed. (A marksmen will not shoot himself!) Treat this duel as a Markov chain by taking as the states the men who are currently alive. Thus the states are:
S_1: All three alive.
S_2: A only alive.
S_3: B only alive.
S_4: C only alive.
S_5: A, B only alive.
S_6: B, C only alive.
S_7: A, C only alive.
S_8: All three dead.
The timesteps will be rounds of the duel. Note: since person A will not shoot himself, the duel will end if it reaches S_2. Treat this as an *absorbing state*, i.e., $p_{22} = 1$ and $p_{2j} = 0$ for all $j \neq 2$. Likewise for S_3, S_4, and S_8.

(a) Draw a state diagram for this Markov Chain and find the transition matrix P.

(b) Assuming at the start that all three men are alive, find the probability that all of them are dead prior to round 5.

(c) Assuming at the start all three men are alive, find the probability that person C is dead prior to round 5.