MATH 209: Homework 3 – due Friday, 2/9 at 2pm on gradescope

(P1) Based on historical data, it has been determined that the demand for spring rolls at the *Thai House Super Buffet* on Friday nights is well-modeled by a normal distribution. On average, 292 spring rolls are consumed each Friday night, with variance of 4900. Moreover,

- Rolls are made fresh each day, and any left over are thrown out at the end of the night. The cost of making each roll is estimated as \$0.53.
- If the Thai House runs out of rolls, they need to call-in an emergency cook to prepare new ones on the spot. This process is estimated to cost \$1.84 per roll because emergency cooks are paid at a higher rate. Management insists that every customer wanting a roll, must get one.

Find the optimal stocking level s^*. How many standard deviations of safety stock should you carry? Note: you can find many calculators online to get a a numerical value for $\Phi^{-1}(v)$ for a given v. For instance, Microsoft Excel has a command for this function.

(P2) Consider the basic newsvendor model as discussed in section 1.4.

(a) Show that if the demand is normally distributed, one should carry no safety stock when $b = h$ (i.e. $s^* = \mu$), a positive amount of safety stock if $b > h$, and a negative amount of safety stock if $b < h$.

(b) Suppose $b = h$ but now the demand is exponentially distributed with mean μ. Find s^*. Is there a positive amount of safety stock (i.e. $s^* > \mu$), a negative amount, or none?

(P3) Suppose demand D for a product over a week is Erlang-2 distributed with mean μ.

(a) Integrate the density of D to show that the cumulative distribution function is

$$P(D \leq x) = 1 - e^{-2x/\mu}\left(1 + \frac{2x}{\mu}\right), \text{ for } x \geq 0.$$

(b) Now suppose $\mu = 20$, and that back order and holding costs per unit are equal. Find the optimal stocking level s^* in the newsvendor model. Note: you will need to numerically solve for s^* using a root solver of some kind.