MATH 208: Written Homework 7 – due Friday, 11/1 at 2pm on gradescope

(P1) This question refers to problem P3 from HW5:

"Farmer Alice has 150 acres on which she can grow corn or broccoli and $ 1950 available for labor and upfront expenses. Each acre of corn requires $30 in upfront costs and uses 15 hours of labor. Each acre of broccoli requires $21 in upfront costs and uses 9 hours of labor. Labor is $18 per hour. The sale of corn yields $600 per acre and the sale of broccoli yields $240 per acre."

The LOP to maximize profit by deciding how much of each vegetable to plant is given by

$$\begin{aligned} \text{Maximize} \quad & z = 300x_1 + 57x_2, \\ \text{such that} \quad & x_1 + x_2 \leq 150, \\ & 300x_1 + 183x_2 \leq 1950, \\ \text{and} \quad & x_1, x_2 \geq 0, \end{aligned}$$

and the final tableau is given by

$$\left[\begin{array}{cc|ccc|c} 0 & 117 & 300 & -1 & 0 & 43050 \\ 300 & 183 & 0 & 1 & 0 & 1950 \\ \hline 0 & 37800 & 0 & 300 & 300 & 585000 \end{array}\right].$$

Answer the following.

(a) How much less profitable must corn become so that Alice begins converting corn acreage to broccoli? Assume broccoli profits remain fixed.

(b) How much more profitable must broccoli become so that Alice begins converting corn acreage to broccoli? Assume corn profits remain fixed.

(P2) Referring to the previous problem, draw a graph that shows the region of corn and broccoli profits per acre in which Alice makes no crop changes. Note: in other words, determine a region in the c_1, c_2 plane, where $z = c_1x_1 + c_2x_2$. Hint: Consider $z = (300 + \delta_1)x_1 + (57 + \delta_2)x_2$.

(P3) Consider the following ILP (call it P):

$$\begin{aligned} \text{Max } z &= 10x_1 + 8x_2 \\ \text{such that } 11x_1 + 5x_2 &\leq 36 \\ 5x_1 + 9x_2 &\leq 34 \\ \text{and } x_1, x_2 &\in \mathbb{N} \end{aligned}$$

Use the branch-and-bound algorithm to find the ILP optimum of P. Draw the branching tree and fully explain how your results in the tree allow you to conclude that you have found the optimum.