## Homework 9 Math 202 Stochastic Processes Spring 2024

Question 1. Let $X_{t}$ be a continuous-time birth and death process with birth rate $\lambda_{n}=1+\frac{1}{n+1}$ and death rate $\mu_{n}=1$. Is this process positive recurrent, null recurrent or transient? What if $\lambda_{n}=1-\frac{1}{n+2}$ ?

Question 2. Consider the population model with immigration. For which values of $\mu, \lambda, \nu$ is the chain positive recurrent, null recurrent or transient?

Question 3. Consider a birth and death process with $\lambda_{n}=\frac{1}{n+1}$ and $\mu_{n}=1$. Show that the process is positive recurrent and give the stationary distribution.

Question 4. Consider the experiment of rolling two dice. Let $X$ be the value of the first roll and $Y$ the sum of the two dice. Find $\mathrm{E}[X \mid Y]$, i.e give the value of $\mathrm{E}[X \mid Y](y)$ for all $y$.

Question 5. Suppose that $X_{t}$ is a Poisson pricess with parameter $\lambda=1$. Find $\mathrm{E}\left[X_{1} \mid X_{2}\right]$ and $\mathrm{E}\left[X_{2} \mid X_{1}\right]$.

