Homework 9

Math 202 Stochastic Processes Spring 2024

Question 1. Let X_t be a continuous-time birth and death process with birth rate $\lambda_n = 1 + \frac{1}{n+1}$ and death rate $\mu_n = 1$. Is this process positive recurrent, null recurrent or transient? What if $\lambda_n = 1 - \frac{1}{n+2}$?

Question 2. Consider the population model with immigration. For which values of μ , λ , ν is the chain positive recurrent, null recurrent or transient?

Question 3. Consider a birth and death process with $\lambda_n = \frac{1}{n+1}$ and $\mu_n = 1$. Show that the process is positive recurrent and give the stationary distribution.

Question 4. Consider the experiment of rolling two dice. Let X be the value of the first roll and Y the sum of the two dice. Find E[X|Y], i.e give the value of E[X|Y](y) for all y.

Question 5. Suppose that X_t is a Poisson pricess with parameter $\lambda = 1$. Find $\mathbb{E}[X_1|X_2]$ and $\mathbb{E}[X_2|X_1]$.