

## Homework 8

### Math 202 Stochastic Processes Spring 2024

**Question 1.** *Shocks occur to a system according to a Poisson process of rate  $\lambda$ . Suppose that the system survives each shock with probability  $\alpha$ ,  $0 < \alpha < 1$  independently of other shocks, so that its probability of surviving  $k$  shocks is  $\alpha^k$ . What is the probability that the system is surviving at time  $t$ ? What happens to this probability as  $t \rightarrow \infty$ ?*

**Question 2.** Let  $X_t$  be a Markov process with state space  $\{1, 2\}$  and rates  $\alpha(1, 2) = 1$ ,  $\alpha(2, 1) = 4$ . Find  $P_t$ .

UR MATH 202

**Question 3.** Consider the continuous time Markov chain with state  $\{1, 2, 3, 4\}$  and the infinitesimal generator

$$A = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 0 & -3 & 2 & 1 \\ 1 & 2 & -4 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

- (a) Find the equilibrium distribution.
- (b) Suppose the chain starts at 1. What is the expected amount of time until it changes the state for the first time?
- (c) Suppose the chain starts at 1. What is the expected amount of time until the chain is in state 4?

**Question 4.** Consider the continuous time Markov chain with state 1, 2, 3, 4 and the infinitesimal generator

$$A = \begin{bmatrix} -2 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 1 & -3 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

- (a) Find the equilibrium distribution.
- (b) Suppose the chain starts at 1. What is the expected amount of time until it changes the state for the first time?
- (c) Suppose the chain starts at 1. What is the expected amount of time until the chain is in state 4?