Homework 7

Math 202 Stochastic Processes Spring 2024

Question 1. *For each $h > 0$, let $X(h)$ have a Poisson distribution with parameter λh. Let $p_k(h) = P(X(h) = k)$ for $k = 0, 1, \ldots$ Verify that*

$$\lim_{h \to 0} \frac{1 - p_0(h)}{h} = \lambda, \text{ or } p_0(h) = 1 - \lambda h + o(h)$$

$$\lim_{h \to 0} \frac{p_1(h)}{h} = \lambda, \text{ or } p_1(h) = \lambda h + o(h)$$

$$\lim_{h \to 0} \frac{p_2(h)}{h} = 0, \text{ or } p_2(h) = o(h)$$

Question 2. *Customers arrive at a service facility according to a Poisson process of rate λ customers/hour. Let $N(t)$ be the number of customers that have arrived up to time t. Let $T_1, T_2, \ldots$ be the successive arrival times of the customers.*

(a) *Determine the conditional mean $E[T_1 | N(t) = 2]$.*

(b) *Determine the conditional mean $E[T_3 | N(t) = 5]$.*
(Hint: It might be helpful to notice that for $U \sim$ Uniform$[0, t]$, $t - U \sim$ Uniform$[0, t]$.)

(c) *Determine the conditional probability density function for T_2 , given that $N(t) = 5$.*

Question 3. *Suppose that the number of calls per hour arriving at an answering service follows a Poisson process with intensity* $\lambda = 4$ *per hour.*

(a) *What is the probability that fewer than two calls come in the first hour?*

(b) *Suppose that six calls arrive in the first hour. What is the probability that at least two calls will arrive in the second hour?*

(c) *The person answering the phones waits until fifteen phone calls have arrived before going to lunch. What is the expected amount of time that the person will wait?*

(d) *Suppose it is known that exactly eight calls arrived in the first two hours. What is the probability that exactly 5 of them arrived in the first hour?*

(e) *Suppose it is known that exactly* k *calls arrived in the first four hours. What is the probability that exactly* j *of them arrived in the first hour?*

Question 4. *Let X_t and Y_t be two independent Poisson processes with rates λ_1 and λ_2, respectively, measuring number of customers arriving in stores 1 and 2, respectively.*

(a) What is the probability that a customer arrives in store 1 before any customers arrive in store 2?

(b) What is the probability that in the first hour, a total of exactly four customers arrive in store 2?

(c) Given that exactly four customers have arrived at the two stores, what is the probability that all 4 went to store 1?

(d) Let T denote the time of arrival of the fist customer at store 2. Then X_T is the number of customers in store 1 at the time of the first customer arrival at store 2. Find the probability distribution of X_T.