Homework 6

Math 202 Stochastic Processes Spring 2024

Question 1. *Consider the Markov chain described in HW3 Q2.*

(a) *After a long time, what would be the expected number of papers in the pile?*

(b) *Assume the pile starts with 0 papers. What is the expected time until the pile will again have 0 papers?*

Question 2. *Suppose we flip a fair coin repeatedly until we have flipped four consecutive heads. What is the expected number of flips that are needed? (Hint: consider a Markov chain with state space* $\{0, 1, 2, 3, 4\}$*.)*

Question 3. *Consider the Markov chain with state space* $S = \{0, 1, 2\cdots\}$ *and transition probabilities:*

$$p(x, x+1) = 2/3; \;\; p(x, 0) = 1/3.$$

Show that the chain is positive recurrent and give the limiting probability π.

Question 4. *Consider the simple ($p = q = 1/2$) random walk on integers $\mathbb{Z}$. We argued in class that*

$$p_n(0,0) \sim \frac{C}{\sqrt{n}}$$

for some constant C.

(a) *Show that any two states i and j communicate.*

(b) *Show that 0 is recurrent by showing that the sum $\sum_n p_n(0,0)$ is divergent (consider the integral test for series)*

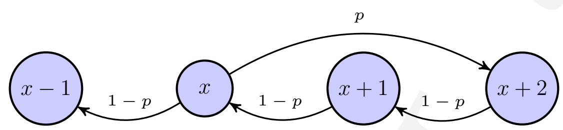
(c) *Is the symmetric random walk null recurrent or positive recurrent?*

Question 5. *Consider the Markov chain with state space $S = \{0, 1, 2 \cdots\}$ and transition probabilities:*

$$p(x, x+2) = p; \;\; p(x, x-1) = 1-p \;\; x > 0$$
$$p(0,2) = p, \;\; p(0,0) = 1-p.$$

For which values of p is this a transient chain? Hint: Use the Stirling's approximation:

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$



Question 6. *Consider the queueing model. For which values of p, q is the chain null recurrent, positive recurrent, transient? For the positive recurrent case give the limiting probability distribution π. What is the average length of the queue in equilibrium? For the transient case, give $\alpha(x)$ = the probability starting at x of ever reaching at 0.*