Homework 5

Math 202 Stochastic Processes Spring 2024

Question 1. *A coin is tossed repeatedly until two successive heads appear. Find the mean number of tosses required.*

(a) Define a Markov chain (let X_n be the MC that represents the current number of successive heads that have appeared) and find its transition matrix.

(b) Condition on the first-step and compute the mean number of tosses required.

Question 2. *An urn contains five tags, of which three are red and two are green. A tag is randomly selected from the urn and replaced with a tag of the opposite color. This continues until only tags of a single color remain in the urn. Let X_n denote the number of red tags in the urn after the nth draw, with $X_0 = 3$. What is the probability that the game ends with the urn containing only red tags?*

Question 3. *Consider the random walk Markov chain whose transition probability matrix is given by*

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.3 & 0 & 0.7 & 0 \\ 0 & 0.3 & 0 & 0.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Starting in state 1 *determine the mean time to absorption.*

Question 4. *A Markov chain has the transition probability matrix*

$$P = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.5 & 0.1 & 0.4 \\ 0 & 0 & 1 \end{bmatrix}$$

with states labeled $\{0, 1, 2\}$*. It is known to start in state* $X_0 = 0$*. Eventually, the process will end up in state 2. What is the probability that the time* $T = \min\{n \geq 0; X_n = 2\}$ *is an odd number?*

Question 5. *Consider the random walk with probabilities p and $q = 1-p$ of going forwards or backwards on the state space $\{1, 2, 3, 4\}$. The states 1 and 4 are absorbing.*

(a) *What are the communication classes?*

(b) *What is the periodicity of each communication class?*

(c) *Let T be a transient class in this system. Show that $p_n(i,j) \to 0$ for any i, j in the communication class T.*