Homework 2
Math 202 Stochastic Processes Spring 2024

Question 1. *Let N cards carry the distinct numbers $x_1, \ldots, x_N$. If two cards are drawn at random without replacement, show that the correlation coefficient ρ between the numbers appearing on the two cards in $-1/(N-1)$.*

{If this questions looks too abstract, you can assume the numbers are $1, 2, \cdots, N$ and start doing the computation for $N = 5$, then try to generalize.}

Question 2. *Let U, V, W be independent random variables with equal variances σ^2. Let $X = U + V$ and let $Y = V - W$. Find the covariance of X and Y.*

Question 3. *Find all functions $x(t)$, $y(t)$ so that $x'(t) = 5x - y$, $y'(t) = 3x + y$ Find the particular solution with initial position $(x(0), y(0)) = (1, 3)$.*

Question 4. *Find all functions f from integers to complex numbers so that*

$$f(n+1) = 4f(n) - 5f(n-1).$$

Now find the solution when $f(0) = f(1) = 2$ and explain why it is real.

Question 5. *Find the function $f(n)$ so that $f(0) = 0$*

$$f(n) = \frac{1}{3}[f(n-1) + f(n+1) + f(n+2)], \qquad n \geq 1$$

and

$$\lim_{n \to \infty} f(n) = 1.$$