Homework 10

Math 202 Stochastic Processes Spring 2024

Question 1. *Let X and Y be independent Poisson random variables with respective parameters λ and μ. Let $S = X + Y$. Find the conditional expectation of Y given $S = n$ and then find $E[Y|S]$.*

Question 2. *Assume that* X *and* Y *have joint density*

$$f(x,y) = \frac{2}{xy}, \text{ for } 1 < y < x < e.$$

Find $E[Y|X]$.

Question 3. *Ellen's insurance will pay for a medical expense subject to a \$100 deductible. Assume that the amount of the expense is exponentially distributed with mean \$500. Find the expectation and standard deviation of the payout. Hint: Let X be the insurance company's payout,then*

$$X = \begin{cases} M - 100, & \text{if } M > 100 \\ 0, & \text{if } M \leq 100 \end{cases}$$

where M is the amount of medical expense. Use tower property of conditional expectation.

Question 4. *Consider independent random variables X, Y, and U, where U is uniformly distributed on $(0,1)$ and $E[X^2] = \sigma_X^2$ and $E[Y^2] = \sigma_Y^2$. Find the conditional expectation*

$$E\left(UX^2 + (1-U)Y^2 \middle| U\right)$$

Question 5. *Let $X_1, X_2, \cdots$ be i.i.d random variables. Let $m(t) = \mathrm{E}\left[e^{tX_1}\right]$ be the moment generating function of X_1 (and hence of each X_i). Fix t and assume $m(t) < \infty$. Let $S_0 = 0$ and for $n > 0$,*

$$S_n = X_1 + \cdots + X_n.$$

Let $M_n = m(t)^{-n} e^{tS_n}$. Show that M_n is a martingale with respect to $\{\mathcal{F}_n\}_n$ where $\mathcal{F}_n = \sigma(X_i : i \leq n)$, i.e. the information contained in $X_1, \cdots, X_n$.