Homework 1

Math 202 Stochastic Processes Spring 2024

Question 1. *(a) Plot the distribution function:*

$$F(x) = \begin{cases} 0 & x \leq 0 \\ x^3 & 0 < x < 1 \\ 1 & x \geq 1 \end{cases}$$

(b) Determine the corresponding density function $f(x)$ in the three regions.

(c) What is the mean of the distribution?

(d) If X is a random variable with distribution F, then evaluate $P(1/4 \leq X \leq 3/4)$.

Question 2. *Determine the distribution function, mean and variance corresponding to the triangular density:*

$$f(x) = \begin{cases} x & 0 \leq x \leq 1, \\ 2 - x & 1 \leq x \leq 2, \\ 0 & \textit{otherwise} \end{cases}$$

Question 3. *Let* 1_A *be the indicator random variable associated with an event* A*, defined to be one if* A *occurs, and zero otherwise. Show*

(a) $1_{A^c} = 1 - 1_A$

(b) $1_{A \cap B} = 1_A 1_B = \min(1_A, 1_B)$

(c) $1_{A \cup B} = \max(1_A, 1_B)$.

Question 4. *Let X and Y be independent random variables having distribution F_X and F_Y respectively.*

(a) *Let $Z = \max(X, Y)$. Express $F_Z(t)$ in terms of $F_X(s)$ and $F_Y(u)$.*

(b) *Let $W = \min(X, Y)$. Express $F_W(t)$ in terms of $F_X(s)$ and $F_Y(u)$.*

Question 5. *Let U have a Poisson distribution with parameter λ and let $V = 1/(1+U)$. Find the expected value of V.*