

# Worksheet 9

1. Let  $X \sim \text{Exp}(2)$ . Find a real number  $a < 1$  so that the events  $\{X \in [0, 1]\}$  and  $\{X \in [a, 2]\}$  are independent.
2. An urn has 2 red, 5 white, and 3 green balls. Select 3 balls at random and let  $X$  be the number of red balls and  $Y$  the number of white balls.
  - (a) Determine the joint p.m.f. of  $(X, Y)$ .
  - (b) Determine the marginal/individual p.m.f.'s for  $X$  and  $Y$ .
  - (c) Compute  $P(X \geq Y)$ .
  - (d) Compute  $P(X = 2|X \geq Y)$ .
3. Suppose  $X, Y$  have joint density function

$$f(x, y) = \begin{cases} \frac{12}{7} (xy + y^2), & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0, & \text{else.} \end{cases}$$

- (a) Check that  $f$  is a genuine joint density function.
- (b) Find the marginal/individual density functions of  $X$  and  $Y$ .
- (c) Calculate the probability  $P(X < Y)$ .
- (d) Calculate the expectation  $E[X^2Y]$ .