

Worksheet 8

1. A random variable X has density function

$$f(x) = \begin{cases} c(x + x^2), & x \in [0, 1], \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Determine c .
- (b) Compute $E(1/X)$.
- (c) Determine the probability density function of $Y = X^2$.
2. A political interest group wants to determine what fraction $p \in (0, 1)$ of the population intends to vote for candidate A in the next election. 1000 randomly chosen individuals are polled. 457 of these indicate that they intend to vote for candidate A. Find the 95% confidence interval for the true fraction p .
3. A marketing firm wants to determine what proportion $p \in [0, 1]$ of targeted customers prefer strawberries to blueberries. They poll n randomly chosen customers and discover that X of them prefer strawberries. How large should n be in order to know with at least 0.9 certainty that the true p is within 0.1 of the estimate X/n ?
4. A hockey player scores at least one goal in roughly half of his games. How would you estimate the percentage of games where he scores a hat-trick (three goals)? Hint: Let the number of goals that a Hockey player scores in one game be modeled by the Poisson distribution.
5. Suppose you need to take a bus to get to class in the morning. You notice that, on average, 4 buses stop outside your home every hour.
- (a) Assume that the number of buses that arrive each hour is Poisson distributed. What is the probability that only 2 buses will arrive in the next hour?
- (b) You look out your window and see that a bus just left the bus stop. You head downstairs to catch the next bus and arrive at the bus stop 5 minutes after the last bus departed? What is the probability that you will have to wait more than 10 minutes to catch the next bus?
Hint: If the number of buses that arrive in an hour is Poisson(λ), then the waiting time between buses is Exp(λ).