Math 201

Spring 2022

Worksheet 7

1. Suppose that the random variable X has expected value $E[X] = 3$ and variance $\text{Var}(X) = 4$. Compute the following quantities:
   (a) $E[3X + 2]$
   (b) $E[X^2]$
   (c) $E[(2X + 3)^2]$
   (d) $\text{Var}(4X - 2)$

2. Let X be a normal random variable with mean 3 and variance 4.
   (a) Find the probability $P(2 < X < 6)$.
   (b) Find the value c such that $P(X > c) = 0.33$.
   (c) Find $E[X^2]$. *Hint:* It is quicker to use properties of the mean and variance than to integrate the density function.

3. Let $Z \sim \mathcal{N}(0, 1)$. Show that the n-th moments of Z, m_n $(n \geq 1)$ are given by

$$m_n = \begin{cases} 0 & \text{odd } n \\ (n-1)!! & \text{even } n, \end{cases}$$

where $(n-1)!! = (n-1)(n-3)(n-5) \cdot \ldots \cdot 1$. *Hint:* Show that, for even n, $m_n = (n-1)m_{n-2}$.

(Bonus) A random variable X is said to have the Poisson Distribution with parameter $\lambda > 0$ if its probability mass function is given by

$$P(X = k) = \frac{\lambda^k}{k!}e^{-\lambda} \qquad k \in \{0, 1, 2, 3, \ldots\}.$$

   (a) Show that $E[X] = \lambda$. It may be helpful to recall that

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}.$$

   (b) Show that $Var(X) = \lambda$. It will be helpful to recall that k=(k-1)+1.