Math 201

Spring 2022

Worksheet 5

1. A fair coin is flipped 3 times. Let X be the number of heads observed.
   (a) Give both the range and probability mass function for X.
   (b) Find $P(X \leq 1)$ and $P(X > 1)$.
   (c) Find the CDF of X.

2. Suppose that the random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 0, & \text{if} \quad x < \sqrt{2}, \\ x^2 - 2, & \text{if} \quad \sqrt{2} \leq x < \sqrt{3} \\ 1 & \text{if} \quad \sqrt{3} \leq x. \end{cases}$$

   (a) Find the range of X.
   (b) Find $P(X = 1.6)$.
   (c) Find $P(1 \leq X \leq 3/2)$.
   (d) The random variable X has a probability density function. Find it!

3. Recall that if $X \sim Geom(p)$ then $P(X = n + k | X > n) = P(X = k)$, for every $n, k \geq 1$. This one of the ways to define the memoryless property of the geometric distribution. It states the following: given that there are no successes in the first n trials, the probability that the first success comes at trial $n + k$ is the same as the probability that a freshly started sequence of trials yields the first success at trial k.

   Show if X is any random variable with range $\mathbb{N} = \{1, 2, 3, ...\}$ that satisfies $0 < P(X = 1) < 1$, and the memoryless property in (a), then X must be a geometric random variable for some parameter value $p \in (0, 1)$. In other words, the geometric distribution is the unique discrete distribution on the positive integers with the memoryless property.
   Hint. Use $P(X = k) = P(X = k + 1 | X > 1)$ repeatedly.