Math 201

Spring 2022

Worksheet 4

1. There are two boxes, each containing one marble. A marble is either red or green with probability 50% each.

   (a) What is the probability that both marbles are green?

   (b) Someone randomly chooses one of the boxes and opens it. What is the probability that both marbles are green if the revealed marble is green?

   (c) Instead of choosing randomly, the person first peeks into both boxes, then opens one using the policy, that if any of the boxes contains a green one, then he always chooses to reveal a green marble. What is the probability that both marbles are green if the revealed marble is green?

2. The Acme insurance company has two types of customers, careful and reckless. A careful customer has an accident during the year with probability 0.01. A reckless customer has an accident during the year with probability 0.04. 80% of the customers are careful and 20% of the customers are reckless. Suppose a randomly chosen customer has an accident this year. What is the probability that this customer is one of the careful customers?

3. Urn A contains 2 red and 4 white balls, and urn B contains 1 red and 1 white ball. A ball is randomly chosen from urn A and put into urn B, and a ball is then chosen from urn B. What is the conditional probability that the transferred ball was white given that a white ball is selected from urn B?

4. Suppose that a person's birthday is a uniformly random choice from the 365 days of a year (leap years are ignored), and one person's birthday is independent of the birthdays of other people. Alex, Betty and Conlin are comparing birthdays. Define these three events:

   $$A = \{\text{Alex and Betty have the same birthday}\}$$
   $$B = \{\text{Betty and Conlin have the same birthday}\}$$
   $$C = \{\text{Conlin and Alex have the same birthday}\}$$

   Are these events independent?