Math 201

Spring 2022

Worksheet 3

1. An urn contains b red and c green balls. One ball is drawn randomly from the urn and its color observed. It is then returned to the urn and an additional r balls of the same color are added to the urn. Next the selection process is repeated. Prove by induction that the probability to choose a red ball at any stage is $\frac{b}{b+c}$.

2. (a) A random variable X is degenerate if there is some real number b such that $P(X = b) = 1$. Construct an example of a random variable that is degenerate but not a constant function on the sample space Ω. Explicitly, define a sample space Ω, a probability measure P on Ω and a random variable X such that there exists a real number b with $P(X = b) = 1$, but $X(\omega)$ is not constant for all $\omega \in \Omega$.

   (b) Let Y be a random variable which takes values on the integers. Show that for every integer k we have

   $$P(Y = k) = P(Y \leq k) - P(Y \leq k - 1).$$

   Hint: One way to do this would be to use induction, but I don't expect any proofs from you. So as long as you get the idea in some reasonable way.

3. Consider the following experiment: flip a coin and throw a die. Define a probability space, and a probability measure. Define a random variable X that is 1 if heads shows up and 2 if tails shows up. Define a random variable Y that represents that number on the die. Define a random variable $T = \min(X, Y)$ and find its probability mass function.