

Worksheet 11

1. I have an urn with 30 chips numbered from 1 to 30. The chips are then selected one by one, without replacement, until all 30 have been selected. Let X_i denote the value of the i -th pick. Find

$$E(X_3 + X_{10} + X_{22}).$$

Hint: All of these variables have the same probability mass functions.

2. Let (X, Y) be a uniformly distributed random point on the quadrilateral D with vertices $(0, 0)$, $(2, 0)$, $(1, 1)$, and $(0, 1)$. Calculate the covariance of X and Y . Based on the description of the experiment, should it be negative or positive?
3. Suppose that for random variables X, Y we have $E[X] = 2$, $E[Y] = 1$, $E[X^2] = 5$, $E[Y^2] = 10$ and $E[XY] = 1$.

(a) Compute $\text{Corr}(X, Y)$.

(b) Find a number c so that X and $X + cY$ are uncorrelated.

4. I roll a fair die four times. Let X be the number of different outcomes that I see. (For example, if the die rolls are 5, 3, 6, 6 then $X = 3$ because there are 3 different outcomes.)

Hint: Use indicator random variables.

(a) Find the mean of X .

(b) Find the variance of X .