Math 201, Spring 2022

Problem Set # 7

Due March 23, 2022 at 11:59pm on gradescope

Question 1. A pollster would like to estimate the fraction p of people in a population who intend to vote for a particular candidate. How large must a random sample be in order to be at least 95% certain that the fraction $\hat{p}$ of positive answers in the sample is within 0.02 of the true p?

Solution. We use the formula

$$P(|\hat{p} - p| < \epsilon) \geq 2\Phi(2\epsilon\sqrt{n}) - 1.$$

For a 95% confidence interval with $\epsilon = 0.02$ we have

$$\begin{aligned} 2\Phi(2\epsilon\sqrt{n}) - 1 &\geq .95 \\ \Phi(2\epsilon\sqrt{n}) &\geq .975. \end{aligned}$$

We have $\Phi(1.96) = .9750$ using the table in Appendix E thus

$$2\epsilon\sqrt{n} = 1.96$$

and with $\epsilon = .02$ we find $n = 2401$.

Question 2. Approximate the probability that out of 300 die rolls we get exactly 100 numbers that are multiples of 3. Hint. You will need the continuity correction for this.

Solution For each die roll, there is a 1/3 chance of getting a multiple of 3 (rolling a 3 or 6). Let X be the number of multiples of 3 in 300 die rolls, so $X \sim Bin(300, 1/3)$. Considering $\text{Var}(X) = np(1-p) = 300(1/3)(2/3) = 200/3 > 66$, a normal approximation is appropriate. Below $Z \sim N(0, 1)$:

$$\begin{aligned} \mathbb{P}(X = 100) &= \mathbb{P}(99.5 \leq X \leq 100.5) \\ &= \mathbb{P}\left(\frac{99.5 - 100}{\sqrt{200/3}} \leq \frac{X - np}{\sqrt{np(1-p)}} \leq \frac{100.5 - 100}{\sqrt{200/3}} \right) \\ &\approx \mathbb{P}\left(-0.0612 \leq \frac{X - 100}{\sqrt{200/3}} \leq 0.0612 \right) \\ &\approx \mathbb{P}\left(-0.0612 \leq Z \leq 0.0612 \right) \\ &= \Phi(0.0612) - \Phi(-0.0612) \\ &= \Phi(0.0612) - (1 - \Phi(0.0612)) \\ &= 2\Phi(0.0612) - 1 \approx 2(0.5244) - 1 = 0.0488 \end{aligned}$$