

Math 201, Spring 2022

Problem Set # 6

Due March 16, 2022 at 11:59pm on gradescope

Question 1. Let $Z \sim \mathcal{N}(0, 1)$ and $X \sim \mathcal{N}(\mu, \sigma^2)$. This means that Z is a standard normal random variable with mean 0 and variance 1, while X is a normal random variable with mean μ and variance σ^2 .

- a) Calculate $E[Z^3]$ using integration by parts (as in the proof of Fact 3.59 in the book).
- b) Calculate $E[X^3]$. **Hint:** Do not integrate with the density function of X unless you like messy integration. Instead use the fact that $X = \sigma Z + \mu$ and expand the cube inside the expectation.

Question 2. My bus is scheduled to depart at noon. However, we're in Randomland, and this means the departure time varies randomly with average departure time 12 o'clock and a standard deviation of 6 minutes. Assume the departure time is normally distributed. If I get to the bus stop 5 minutes past noon, what is the probability that the bus has not yet departed?