Math 201, Spring 2022

Problem Set # 4

Due February 16, 2022 at 11:59pm on gradescope

Question 1. Show that if $X \sim \text{Geom}(p)$ then

$$P(X = n + k | X > n) = P(X = k), \text{ for every } n, k \geq 1.$$

This is sometimes called the *memoryless property* of the geometric distribution. It says that if there are no successes in the first n trials then the probability that the first success at trial $n + k$ is the same as the probability that a freshly started sequence of trials yields the first success at trial k. The first n trials are forgotten.

From the definition we have

$$P(X = k) = p(1 - p)^{k-1}.$$

We also have

$$\begin{aligned} P(X = n + k | X > n) &= \frac{P(X = n + k, X > n)}{P(X > n)} \\ &= \frac{p(1-p)^{n+k-1}}{(1-p)^n} \\ &= p(1-p)^{k-1} \end{aligned}$$

therefore

$$P(X = k) = P(X = n + k | X > n).$$

Question 2. Consider the square in the plane consisting of all (x, y) such that $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$, and let Q be a point chosen uniformly at random inside the square. Let X be the distance from Q to $(0, 0)$.

(a) Calculate $P(X > 1)$.

(b) Calculate $P(X < 0.5)$.

Solution. This square is 2×2, so its area is 4. The set of points within it that are at a distance of 1 or less from the origin is simply the circle of radius 1. Thus the area of points Q where that are a distance of more than 1 is $4 - \pi$. This means that

$$P(X > 1) = \frac{4 - \pi}{4}.$$

For part (b), we find that

$$P(X < 0.5) == \frac{.5^2 \pi}{4}$$