Math 201, Spring 2022

Problem Set # 3

Due February 9, 2022 at 11:59pm on gradescope

Question 1. We have two bins. The first bin has 6 blue marbles and 4 yellow marbles. The second bin has 3 blue marbles and 4 yellow marbles. We choose a bin at random and then draw a marble from that bin.

a) If the marble we select is yellow, what is the probability that we chose the first bin?

b) Now suppose we put the yellow marble from (a) back in the bin it was drawn from and then draw a marble from the same bin. This marble is also yellow. With this new information is it more or less likely that we chose the first bin? What is the probability that we chose the first bin now?

Solution

a) Let Y be the event that the marble drawn is yellow, $B1$ then event the marble was drawn from bin 1, and $B2$ from bin 2.

$$P(B1|Y) = \frac{P(Y|B1)P(B1)}{P(Y|B1)P(B1) + P(Y|B2)P(B2)}$$
$$= \frac{\frac{4}{10}\frac{1}{2}}{\frac{4}{10}\frac{1}{2} + \frac{4}{7}\frac{1}{2}} = 7/17 \approx 0.41.$$

b) We must compute $P(B1|YY)$, that is the probability the marble is drawn from bin 1 given that 2 yellows are drawn. Since we have observed two yellows it is more likely that the balls were drawn from the bin with a greater proportion of yellows, that is bin 2. We can compute this probability using Bayes' formula. We have

$$P(B1|YY) = \frac{P(YY|B1)P(B1)}{P(YY|B1)P(B1) + P(YY|B2)P(B2)}$$
$$= \frac{\frac{4}{10}\frac{4}{10}\frac{1}{2}}{\frac{4}{10}\frac{4}{10}\frac{1}{2} + \frac{4}{7}\frac{4}{7}\frac{1}{2}} = 49/149 \approx 0.33.$$

As expected the probability of choosing bin 1 has decreased given this new information.

Question 2. a) A crime has been committed in a town of 100,000 inhabitants. The police are looking for a single perpetrator, believed to live in town. DNA evidence is found on the crime scene. Kevin's DNA matches the DNA recovered from the crime scene. According to DNA experts, the probability that a random person's DNA matches the crime scene DNA is 1 in 10,000. Before the DNA evidence, Kevin was no more likely to be the guilty person than any other person in town. What is the probability that Kevin is guilty after the DNA evidence appeared? You may assume that if the perpetrator's DNA is tested then it will match the crime scene DNA 100% of the time.

Hint: Reason as in example 2.14 in the textbook.

b) Suppose a new method is developed to test DNA. With this new method the probability that a random person's DNA matches the crime scene DNA is now 1 in 50,000. Suppose that with the new method it is confirmed that Kevin's DNA matches the DNA recovered from the crime scene. What is the probability that Kevin is guilty now?

c) Suppose that a witness observed a yellow car fleeing the scene of the crime but could not give any more details about the car. The town has 1,000 yellow cars. In questioning it is revealed that Kevin also drives a yellow car. What is the probability that Kevin is guilty now if the original method for testing DNA is used? What if the new more accurate method is used?

Solution

a) Using Bayes' formula,

$$P(\text{guilty}|\text{DNA match}) = \frac{P(\text{DNA match}|\text{guilty})P(\text{guilty})}{P(\text{DNA match}|\text{guilty})P(\text{guilty}) + P(\text{DNA match}|\text{not guilty})P(\text{not guilty})}$$
$$= \frac{1 \cdot \frac{1}{100000}}{1 \cdot \frac{1}{100000} + \frac{1}{10000} \cdot \left(1 - \frac{1}{100000}\right)} \approx 9.09\%.$$

b) In this case we have

$$P(\text{DNA match}|\text{not guilty}) = \frac{1}{50000}$$

and so

$$P(\text{guilty}|\text{DNA match}) \approx 0.33.$$

c) With this new information we update

$$P(\text{guilty}) = \frac{1}{1000}.$$

With the original method for testing DNA we have

$$P(\text{guilty}|\text{DNA match}) \approx 0.91$$

and with the new method for testing DNA we have

$$P(\text{guilty}|\text{DNA match}) \approx 0.98.$$